UNIVERSITY OF THE WITWATERSRAND

Master of Science

RESEARCH REPORT: A case-study exploration of the effects that context familiarity, as a variable, may have on learners' abilities to solve problems in Mathematical Literacy (ML).

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Declaration

I declare that this research report, with the following title: A case-study exploration of the effects that context familiarity, as a variable, may have on learners' abilities to solve problems in Mathematical Literacy (ML), is my own work and effort, except where indicated by special reference in the text. No part of the report has been submitted for any other degree. Any views expressed in the report are those of the author, and in no way represent those of the University of the Witwatersrand. The report has not been presented to any other university for examination, either in South Africa or internationally.

SIGNED: ___________________

DATE: __________________

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Abstract

This study serves to explore the notion of context familiarity and how it affects the way learners perform in closed and open-ended problems in Mathematical Literacy (ML). The learners' performances in this study are based on how well they were able to do the following: select the relevant data from the given tables; select the appropriate mathematics and execute them with precision; relate the mathematical solution back to the context in order to understand the problem better. The key findings indicate that more familiar contexts provide better opportunities for learners to: select the relevant data from given tables; select and execute the relevant mathematical tools; and relate the mathematical solution back to the context.

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Chapter One

1.1Title

A case-study exploration of the effects that context familiarity, as a variable, may have on learners' abilities to solve problems in Mathematical Literacy (ML).

1.2Aim

This study focuses on the effects that context familiarity may have on how learners respond to contextualised problems in Mathematical Literacy (ML). Therefore, the study is concerned with the possibility that context familiarity affects the way learners approach questions at a range of levels, including those which demand an in-depth knowledge of mathematical concepts applied to a particular situation. These questions may be open-ended, requiring learners to draw upon multiple mathematical skills to support an argument. ML is a newly introduced subject in the Further Education Training (FET) band in South Africa. According to the ML National Curriculum Statement (DOE, 2003) this new subject requires a proficiency in the ability to apply appropriately selected mathematical skills and concepts to solve authentic, real-life problems.

In this study, based on a review of relevant literature, the following aspects were selected to determine whether or not a group of learners (the research subjects in this study) had understood the question, and were subsequently able to answer the question. The aspects selected which are listed below, are supported by the literature and also based on the ML taxonomy, found in the ML Subject Assessment Guidelines (DOE, 2008). These aspects are, therefore, skills that are needed to understand and answer the question as a whole. Below, the research questions for this study are listed, and followed by a brief explanation of each of the aspects of ML-related problem solving that figure within the research questions.

1.3 Research Question

In what way does the context affect how learners approach and deal with two ML problems, which are designed to differ on the context familiarity variable?

Sub Questions

1. How do more/less familiar contexts impact on the way learners engage with:

a) the selection of appropriate information/mathematical tools;
b) the execution of mathematical procedures and
c) sense-making of answers in relation to the context.

2. What do the findings on Question 1 tell us about the impact of contextual familiarity on the integrated application of content-based and contextual understandings?

1.4 Rationale for the Questions and Sub-Questions

The effect of context on the way learners approach and answer problems is pertinent to ML because the subject centrally involves understanding and engaging with a range of real-life contexts through the effective selection and use of relevant mathematical procedures. The ML Subject Assessment Guidelines document states that: "Mathematical Literacy will develop the use of basic mathematical skills in critically analysing situations and creatively solving everyday problems" (DOE,Subject Assessment Guidelines, 2008). The process involved in the selection and use of mathematics to solve everyday problems in context, subsequently, has given rise to the sub-questions, which are also informed by the theory and relevant literature. More detail on this is provided in Chapters 2 and 3.

1.5 ML in South Africa

According to the ML Curriculum and Assessment Policy Statement (CAPS) (DOE, 2003), the twenty first century world is “a world characterised by numbers, numerically based arguments and data represented and misrepresented in a number of different ways”. ML as a subject aims to prepare students to function and engage with this world. The subject is designed to achieve this rhetoric through the use of authentic contexts taken and/or adapted from real-life situations.

The development of ML “competencies” (DOE, Subject Assessment Guidelines, 2008) involves making sense of real-life contexts, through the use of mathematical content (including mathematical tools and thinking). Contexts drive the focus of problem-solving, and for sense of these problem contexts to be made, learners have to understand the mathematics that is needed to solve problems in context. The point of the study is to see if familiar contexts play a role in allowing learners to select the relevant information from the context, making the correct selection of the mathematical tools that are needed, executing appropriate mathematical procedures and relating the answers back to the context for the purpose of sense-making. Through the inclusion of specific ordering of tasks (either familiar context task followed by unfamiliar context task, and vice versa), I am also interested in whether engagement with familiar contexts have some sort of role in the way learners approach unfamiliar contexts. The ML CAPS curriculum makes the following point in relation to the aims of ML:

> “It is unrealistic to expect that in the teaching of ML learners will always be exposed to contexts that are specifically relevant to their lives, and that they will be exposed to all of the contexts that they will one day encounter in the world. Rather, the purpose of this subject is to equip learners with the necessary knowledge and skills to be able to solve problems in any context that they may encounter in daily life and in the workplace, irrespective of whether the context is specifically relevant to their lives or whether the context is familiar” (CAPS, 2010, pg 8).

The Learning Programme Guidelines (DOE, 2008, pg 7) for ML state the following in relation to the purpose of ML:

> "The subject ML should enable the learner to become a self-managing person, a contributing worker and a participating citizen in a developing democracy. Mathematical Literacy will ensure a broadening of the education of the learner that is suited to the modern world, by ensuring that learners are enabled to become:
>
> - self-managing people;
> - contributing workers and
> - Participating citizens" (pg 7)

The rhetoric suggests that the purpose of ML is to develop "competencies" that will allow learners to become full participants in society. The subject thus requires for learners to manage their lives through becoming more mathematically literate, which means being able to deal with a range of contexts in real-life that contain numerical/mathematical information.

Being able to manage oneself requires a flexible understanding of the mathematics needed to solve a problem. This understanding in ML is defined by a need for the context to be understood well enough for an appropriate range of mathematical tools to be selected and then used to solve problems in context. One way of ensuring that contexts are understood is to expose learners to a variety of situations in everyday life that require numerical and spatial skills in order for sense to be made. More "familiar" contexts may be used to illuminate the mathematics that is needed to solve the problem at hand, with a view to then moving to less familiar contexts.

In the use and engagement of the mathematics, learners relate to whatever they have chosen to work with. How well they relate to the problem depends on the progression of their understanding of contexts and the relevant mathematics, through exposure to similar problems. These problems and their respective contexts become more familiar to the learners. The skills obtained while learners are dealing with more familiar problems may then be applied to less familiar contexts. This idea will be explored in the study by testing whether or not order of contexts (familiar and less familiar) has anything to do with the way learners approach the two problems.

The Programme for International Student Assessment (PISA, 2003, pg 25), an international assessment with an ML focus, mentions how in ML, learners are expected to "use and engage with" the mathematics in order to solve general mathematical problems. Furthermore, what is important in PISA in relation to my study is the fact that they define a specific kind of ML progression in terms of moving from 'personal' to 'local' to 'national/international' contexts. This suggests that learners' 'distance' from the context is one way that problems become harder. 'Distance' from the context can be thought of in the same way as familiarity.

Self-regulation is an essential skill that is required in Mathematics and ML. It involves being able to assess one's own ability to conduct skills needed to solve a problem. In the case of this study, self-regulation would involve being able to assess and explain: the selection of relevant data; the conducting of mathematical procedures; the process of reflecting back to the context. Self regulation would be evident in sense-making, as learners need to reflect upon the suitability of the mathematical answer when the situation is considered.

Self-regulation with respect to ML also requires learners to take a critical stance to mathematical arguments that are presented or created by the learners themselves.

> "The teaching and learning of ML should thus provide opportunities to analyse problems and devise ways to work mathematically in solving them. Opportunities to engage mathematically in this way will also assist learners to become astute consumers of the mathematics reflected in the media" (CAPS, 2010).

It is, therefore, essential to create opportunities in the classroom, when learners are exposed to problems that develop critical and analytical thinking.

Critical and analytical skills are related within ML with problems that require higher order thinking. The ML Subject Assessment Guidelines (DOE, 2008b) have classified questions using the following taxonomy:

Thinking Level 1 – Knowing:

Tasks at the knowing level of the Mathematical Literacy taxonomy require learners to:

- *Calculate using the basic operations.*
- *Know and use appropriate vocabulary such as equation, formula, bar graph, pie chart, Cartesian plane, table of values, mean, median, mode.*
- *Know and use formulae such as the area of a rectangle, a triangle and a circle where each of the required dimensions is readily available.*
- *Read information directly from a table (e.g. the time that bus number 1234 departs from the terminal).*

Thinking Level 2 – Applying routine procedures in familiar contexts:

Tasks at the applying routine procedures in familiar contexts level of the Mathematical Literacy taxonomy require learners to:

- *Perform well-known procedures in familiar contexts. Learners know what procedure is required from the way the problem is immediately available to the student.*
- *Solve equations by means of trial and improvement or algebraic processes.*
- *Draw data graphs for provided data.*
- *Draw algebraic graphs for given equations.*
- *Measure dimensions such as length, weight and time using measuring instruments sensitive to levels of accuracy.*

Thinking Level 3 – Applying Multistep Procedures in a Variety of Contexts:

Tasks at the applying multistep procedures in a variety of contexts level of the Mathematical Literacy taxonomy require learners to:

- *Solve problems using well-known procedures. The required procedure is, however, not immediately obvious from the way the problem is posed. Learners will have to decide on the most appropriate procedure to solve the solution to the question and may have to perform one or more preliminary calculations before determining a solution.*
- *Select the most appropriate data from options in a table of values to solve a problem.*
- *Decide on the best way to represent data to create a particular impression.*

Thinking Level 4 – Reasoning and Reflecting:

Tasks at the reasoning and reflecting level of the Mathematical Literacy taxonomy require learners to:

- *Pose and answer questions about what mathematics they require to solve a problem and then to select and use that mathematical content.*
- *Interpret the solution they determine to a problem in the context of the problem and where necessary to adjust the mathematical solution to make sense in the context.*
- *Critique solutions to problems and statements about situations made by others.*
- *Generalise patterns observed in situations, make predictions based on these patterns and/or other evidence and determine conditions that will lead to desired outcomes.* (pg 27 - 28)

This taxonomy is relevant to the study because it identifies aspects that create a basis for the analytical tools that will be used in this study. These aspects of the thinking levels have been used to categorise learners' responses. This is a brief explanation of how the ML taxonomy has been used:

- The selection of relevant data is in line with Thinking Levels 1, 3 and 4. Learners, under Thinking Level (TL) 1, are expected to show their knowledge of the context and content, by selecting appropriate data from the given table of information. Under TL 3, the learners are approaching this question for the first time, and with that in mind, they are expected to select the most appropriate data from the table, which would be considered as a more complex activity. Under TL 4, learners are expected to interpret the question for the purpose selecting the relevant data from the table.

- The conducting of mathematical procedures – Is overtly seen is TL 2 and 3 because some questions require learners to conduct basic, routine procedures (TL2) whilst others require more complex, multi-step procedures (TL3). The last question of both activities requires learners to conduct multi-step procedures that are not predefined. The fact that the question is open-ended allows learners to show their competence in terms of selection of appropriate data and conducting relevant mathematical procedures.

- Reflecting the relevant data back to the context – Is in line with TL 4 because it requires learners to analyse the mathematical content critically. The analysis is then used as the basis for reflecting back on the context.

Problems in context require learners to make decisions about what mathematical methods to use; mathematical ideas need to be expressed effectively in order to communicate an understanding of the problem; integrated knowledge of content and skills need to be applied and then interpreted in order to make sense of the problem.

1.6 Theory and Relevance to the Study

The theory of Realistic Mathematics Education (RME) (Barnes, 2004) is used within this study. It is relevant to this study because it uses the ideas of 'vertical and horizontal mathematisation' (Barnes, 2004). RME was introduced in the Netherlands as an attempt to reform the teaching and learning of mathematics. It was created in order to make mathematics more accessible to students. This was achieved by taking situations that were 'experientially' close to learners' lives and using them as a means of accessing the necessary mathematics needed to solve the problem. Horizontal mathematisation (HM) involves using a context that is familiar to the learners in order to enable them to access the necessary mathematics to solve a problem. Vertical mathematisation (VM) on the other hand, involves being able to work effectively within the realm of mathematics.

In this study HM will be explored in terms of the effect that context familiarity may have on the way learners access the necessary mathematics. Vertical mathematisation is evident when learners work with the mathematics, which needs to occur in the problem solving process before learners relate the mathematical answer back to the context. I explore also if context familiarity has an effect on vertical mathematisation work in this study. Theory on mathematical modelling (Gravemeijer, 1998) has been used to show how learners may relate the mathematics back to the context for the purpose of sense-making, and this too is investigated in relation to context familairity.

The written evidence that learners present in their solutions to the two problems I have used show the 'argumentation' processes (Meaney, 2007) that have taken place. 'Argumentation' presents evidence of learners' thinking in the form of written texts. This proves useful in the study, as written texts are the central data sources that are going to be analysed. The analysis will refer back to learners' argumentation competences in ML that come in the form of written responses.

More details on the theoretical framework of RME will be provided in Chapter 3 – and the research design in Chapter 4.

1.7 Study Model and Location

In order to answer my research question, I designed two tasks that were broadly parallel in terms of the technical mathematical demands set within the tasks. They differed, however, in terms of contextual familiarity. The more familiar context was about the merit/demerit system used at the school where the study was located, and the less familiar context was based on a table showing the number of teachers, schools, and learners per district in the Free State. By setting one group of tasks within the school context (the merit/demerit system), I was able to select a context that was familiar – 'personal' in PISA's terms – to all the learners participating in the study. By setting the parallel version of the task in the national education context, I aimed to choose a context that was somewhat more 'distant' from the learners. Although I am not claiming that it was equally 'unfamiliar' to all participating learners, I chose it as it provided a context which I could be fairly sure would be 'less familiar' than the personal context task. Three broadly parallel questions were set within each context in order to facilitate comparison. More details of the two tasks are given in Chapter 4.

The school where the research was conducted is an urban, private (IEB) school, situated in the south of Gauteng. It is one of the smallest schools in the area, consisting of approximately 300 learners from grades 0 to 12. The learners selected are multi-racial, from generally affluent families. Access to resources is not an issue for the learners, as the school is equipped with a fully-functional media centre, with access to the internet that is made available to the learners during breaks and after school. Generally, parents at the school are co-operative and show interest in tasks that require collaboration/assistance.

The classes are small, containing a maximum of 22 learners. Due to the limited numbers and the nature of the study, which is qualitative, all 16 learners from the Grade 10 Mathematical Literacy class that I taught were selected to take part in the research. This number was manageable for a case study like this, as it required for me to analyse the results in-depth quantitatively and qualitatively using grounded analysis alongside my theoretical tools. These learners were divided into two groups of 8, with one group attempting the more familiar task first and the other group attempting the more unfamiliar task first. Each group contained the same proportion of learners with low, medium and high averages in ML, ensuring that other possible variables that could impact on performance on the two tasks were controlled to some extent at least.

The relevant academic (mathematical) history was used as a strategy to ensure that the groups selected for the study were equally weighted in terms of previous academic progress, represented as an average of their ML results for terms one and two in grade 10. Grade 10 ML learners were selected for the study, as it was useful to work with learners who had little experience in dealing with authentic, contextualised problems in ML, which they would have gained if they were in grade 11 or 12. This helped to ensure that experience in ML is not the factor that is being explored.

1.8 Personal Rationale

Details of this section will be discussed in more detail in the Literature Review Chapter.

I have found in my own practice that learners perform better in tests and tasks that are based on contexts that they can relate to. My experience is reflected in previous research findings. Meaney (2007) states:

> "The context of the task did influence whether students needed to draw upon higher-order thinking at the conceptual or procedural level of mathematical literacy"

The context of the task, according to Meaney, affects the way learners access their conceptual and procedural thinking. There is no mention of familiarity of context as a variable that could possibly affect the way the learners solve a problem. This is going to be explored in this study in more depth.

Meaney's writing was useful because she makes reference to how learners approach different problem contexts; these learners' approaches, when dealing with different contexts, vary in terms of levels of cognitive demand achieved. This study also focuses on the way students approach problems that are based on different contexts, with the one being more familiar than the other. The content, however, remains the name. Meaney shows that contexts taken from their everyday lives enable learners to achieve higher levels of cognitive demand.

The Subject Assessment Guidelines (SAGS) (ibid) for ML puts across the idea that basic mathematical skills need to be situated within authentic, real-life problems. The purpose of ML, according to the SAGS, is to engage learners with real-life problems in different contexts" (Mathematical Literacy Subject Assessment Guidelines, DoE, page 9, 2008). This combined with the aforementioned, relevant points on PISA made me propose the idea of "familiarity" of context, as problem contexts may be relatively more or less familiar to learners. If the "familiarity" of context affects the way learners approach problems in ML, then it is worth discovering the elements of a context that enable, or hinder, learners' abilities to select and apply the mathematical skills that they have been taught, and interpret answers in context. With respect to standardised national assessments which present a range of different contexts, the 'familiarity' of context may impact on learner's performance. The outcome of this study may serve to prepare learners to better deal with a range of contexts.

The significance of this concept of 'familiarity' will help to illuminate what it is about a context that allows learners to approach a problem with certainty of what needs to be done. This may prove to be worthwhile, as I wish to use the findings of this study to improve my practice, and hopefully influence my colleagues to do the same.

The remainder of this study is structured as follows: In Chapter 2, the related literature will be presented. I then go onto presenting the theory that informs this study Chapter 3. In Chapter 4, the research design will be explained in detail, which is followed by the findings and the analysis thereof in Chapter 5. Concluding comments and reflections are presented in Chapter 6.

Chapter Two

Literature Review

2.1 Introduction to Mathematical Literacy in South Africa

According to the Curriculum and Assessment Policy Statement for ML (CAPS, 2010), there are five “key elements” that define ML. These elements are useful for defining concepts that arise later in the study. The CAPS (2010) states that it represents a ‘repackaging’ of the 2006 National Curriculum Statement (NCS). It retains the same aims, definition and orientation, but spells these out in more detail. Although the CAPS ML curriculum was not in place during the time that the data was collected (2009), its more detailed specification provided insights into aspects of ML that were pertinent to the study.

In relation to this study, the following aspects of ML, detailed in the CAPS document, helped to create a basis for the research: ML is focussed on using basic mathematical concepts to help make sense of a contextualised problem; the contexts chosen are taken from real-life situations that may or may not be familiar to learners; problems in the subject require learners to make decisions about what mathematical methods to use; mathematical ideas need to be expressed effectively in order to communicate an understanding of the problem; integrated knowledge of content and skills need to be applied in order to make sense of the problem. The five elements, which are paraphrased from the CAPS (2010) with supporting references from relevant literature, read as follows:

1. ***Mathematical Literacy involves the use of elementary mathematical content.***

   The content chosen for the subject consists of basic mathematical concepts and skills that help learners to “make sense of numerically and statistically based scenarios” that they may face in everyday life. The need for learners to work with authentic real-life contexts help individuals become contributing workers and self-managing citizens. With respect to the mathematical content covered, it should not be taught in the absence of context. Steen (2001) would relate this to the idea of basic mathematics in variable contexts by referring to ‘quantitative literacy’ and how working only with the mathematics would not ensure that learners become quantitatively literate.

2. Mathematical Literacy involves real-life contexts.

There is a need in ML for learners to be exposed to authentic and relevant contexts, which relate to everyday life. Learners are encouraged to use mathematical concepts in order to make sense of problems derived from these contexts. The mathematical content in ML is thus seen as a conceptual tool that is used to make sense of contextual problems.

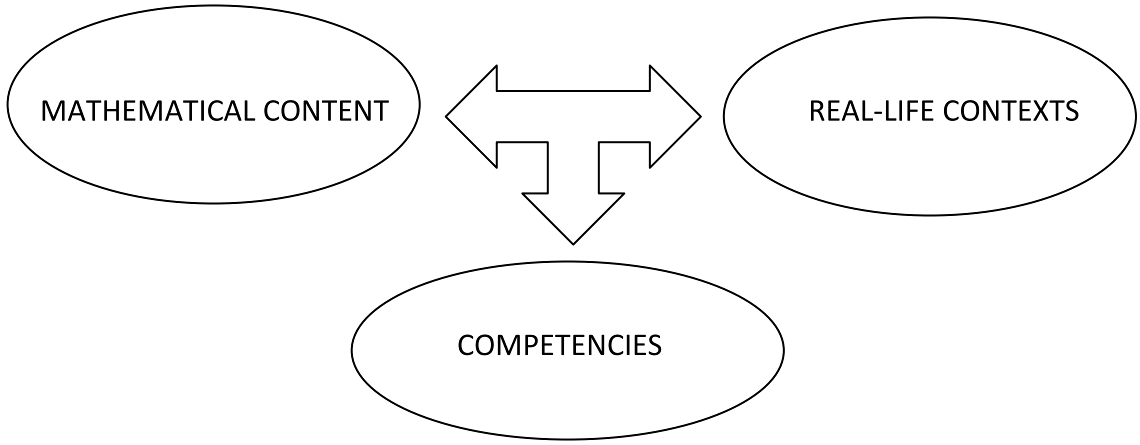
Learners may also draw upon informal, non-mathematical skills to make sense of these problems, as the focus is on making sense of the context and not just knowing how to use the mathematics needed.

3. Mathematical Literacy involves solving familiar and unfamiliar problems.

Learners will be expected to deal with contexts that they are used to, and of course, contexts that may seem foreign to them at this stage of their lives. Therefore, the rhetoric in ML is to equip learners with the necessary knowledge and skills needed to solve problems in all kinds of contexts.

The CAPS argues that learners who are “mathematically literate should have the capacity and confidence to interpret any real-life context that they encounter...” – an important background curriculum aim in relation to my focus on investigating the impact of context familiarity.

Being mathematically literate is defined in the CAPS document by the model shown below (pg. 8):



The model defines being mathematically literate as having universal competencies such as drawing graphs, making comparisons and analysing, which are developed through constant engagement with real-life contexts, and the mathematical content that serves to make sense of the problem-contexts.

4. ***Mathematical Literacy involves decision making and communication.***

This aspect is important for educators to determine whether or not learners have fully understood the problem and context. Learners may be able to make sense of the problem, but if it is not communicated effectively, then the educator may misunderstand the learner completely. The communication of sense-making needs to be clear, and aided by the use of the mathematical tools that have aided the learner's decision making process while referring back to the context.

5. ***Mathematical Literacy involves the use of integrated content and/or skills in solving problems.***

The content in ML is organised according to topics, and this is not the case in real-life. Learners are expected to interpret a problem-context and understand how the content that they have learnt may be of use to them. The next step in ML is to link their understanding of the context to the relevant content needed to make sense of the problem. The position taken in the CAPS document is that after engaging learners in a series of authentic, real-life contexts, it becomes clearer for them to see how the content and acquired skills become essential for sense-making (CAPS, page 8).

The five above-mentioned aspects form the backbone of ML that is central to the study. The first is that mathematical content is needed to make sense of a problem-context. The idea in ML is to ensure that the mathematics is not taught in the absence of the context. In this regard, the aim is that learners are used to applying mathematics to situations that require problems to be solved through mathematical means. The focus in ML is on making sense of real-life, authentic contexts that use mathematical concepts to understand a problem within the context. The contexts covered have to deal with all aspects of life; these may be familiar or unfamiliar to them. 'Competencies' in ML are determined by how well learners are able to apply mathematical content to solve problems across ALL contexts. The competencies are realised through the effective communication (written or verbal) of learners' reasoning. This reasoning needs to clearly display what mathematical and contextual means have been used to make sense of the problem.

Steen (2001) distinguishes between "arithmetic operations", which are basic skills, and "well-founded judgements", which involve the ability to apply basic skills to more complex, contextualised problems. This may help distinguish between richer responses that would contain evidence of higher-order thinking (well-founded judgements). An area I explore in this study is whether such 'well-founded judgements' occur more frequently in more familiar contexts. Steen's aforementioned point with its broad contrast may be used to help distinguish between responses generated from the learners in this study; however it may need more relevant literature to make the process of distinguishing the responses manageable.

With ML being context-driven, learners are required to understand relevant aspects of the focal situation by using the necessary mathematics. This idea created another dimension that has been considered in the literature – the notion of a mathematical and/ or a literacy orientation depending, respectively, on whether the mathematics, or the context, was foregrounded in teachers' questions and explanations – a dimension that is discussed in the literature relating to ML in SA (Venkat, 2007). A mathematical orientation involves focusing primarily on the mathematical procedures, where reference to the context may or may not be present. A literacy orientation entails, mainly, the use of written text to make sense of the problem, where reference to the context is present. In this study, this involves classifying the learner's focus when answering the question, i.e. literacy orientation or mathematical orientation based on what appears to be fore grounded in their answers to the questions set. A literacy orientation contains more text relating to the context/situation, showing that they have tried to make sense of the problem using limited mathematical means. A mathematical orientation shows evidence that the focus is mainly on dealing with the mathematics component of the problem. Some responses could combine both of these elements and show a combined 'mathematical literacy' orientation – in line with curriculum aims.

The idea of 'orientation' provides the study with a factor of influence to consider. In this way, the influence that the *familiarity* of context may have on the way learners make decisions when solving problems can be categorised according to these two different orientations. This idea serves to further elaborate what it means to be mathematically literate, and make sense of open-ended problems in ML.

Focussing on orientation adds additional depth in the analysis chapter. Steen (2001) has stressed that "there are rapidly increasing uses for quantitative literacy in the workplace and in education", it is useful to investigate whether orientation, in addition to solution processes, may be influenced by how familiar a problem context may be to a learner.

Familiarity as a variable has been investigated in a study focused on Physical Science (Alant, 2004), and related there too, to both the content and context. She claims that *familiarity* becomes closely associated with students' experiences of the problem, and in this sense becomes part of their conceptual thinking". Alant's study is focussed on the effect of context *familiarity* in Physical Science. Alant's study refers to the way learners think when they are asked questions with the same content, but derived from different scientific situations. From this case study, Alant has determined that *familiar* contexts/situations enable learners to relate their own experiences to the content, which may become part of their conceptual thinking. This suggests that there are benefits to be gained from exposing ML learners to a wide range of contexts within their ML learning.

Steen (2001) states that quantitatively literate students should "approach problems with confidence in the value of careful reasoning". My experience suggests that the reasoning mentioned is more accessible when learners are more familiar with the context. What is also worth noting is the usefulness of familiar contexts with respect to the way they may be used to help learners grasp mathematical concepts within ML. Venkat, et al (2009) refer to how contexts may provide accessibility to mathematical content. This helps to support the notion that context familiarity could be worth investigating, considering that familiar contexts could potentially provide access to mathematics that would normally be difficult to grasp. Learners may find it easier to communicate an understanding of mathematical content in relation to the situation in, familiar contexts, as the relation to the mathematics (and vice-versa) may occur more naturally. The idea presented here is flagging up the possibility that familiar contexts have often been viewed as effective vehicles for teaching quantitative or mathematical content and procedures.

The term 'numeracy' has also been used to describe the entities related to ML and quantitative literacy (Coben, et al. 2003). Whilst the term Numeracy is sometimes used to refer to the mastery of the basic symbols and processes of arithmetic, in adult skills literature in particular, Numeracy is not only confined to the manipulation of numbers and procedures – it also involves the ability to understand situations through working with numbers.

“Numeracy is often anchored in data derived from and attached to the empirical world” (Steen, 2001). The level of numeracy would involve being able to work with given data, with the ability to reason and connect the mathematics back to the context. According to Steen (2001), “contextual details camouflage broad patterns that are the essence of mathematics”. This brings to mind the idea that completely *unfamiliar* contexts may provide a hindrance for the appropriate mathematical tools to be accessed and worked with, and for sense of a ML problem to be made.

On the other hand, “these same details offer associations that are critically important for long-term learning” (Steen, 2001). Steen refers to how contexts could enable learners to make associations that support a long-term understanding of concepts, which may be used in many ways outside of the classroom. This is pertinent to this research, as the idea of context that Steen has, also helps to provide a basis for the notion of *familiarity*, and its effect on the way learners approach problems in ML. The test used in the study would determine if learners are naturally able to use appropriate mathematical skills, given that the only variable adjusted is *context familiarity.*

A difficulty that Steen has picked up is the idea of ‘compartmentalisation’ (2001), which is when skills and ideas learned in a certain field of study or situation are completely detached from another. One factor that may contribute to this is the *unfamiliarity* of the context. ‘Compartmentalisation’, according to Steen (2001) may prove to be an issue that prevents a learner from allowing their understanding of the mathematics needed to solve a problem in one context enhance their understanding of another, especially if the contexts are parallel in terms of the mathematics that is needed to solve the problem.
PISA (2003) uses the following components to determine whether open-ended questions have been approached and solved holistically:

- correct selection of data for the purpose of understanding the context;
- using relevant mathematics correctly;
- revealing competencies between the content (of mathematics) and the context. (pg. 3)

As noted in chapter one, all of these aspects were identified in the ML taxonomy as well, and are the aspects that I focus on in this study. The study seeks to find a link between *context familiarity* and confidence with being able to apply quantitative methods to contextualised problems in ML. Using the correct data, and applying the relevant mathematical procedures involves interpreting the question to a point where reasoning is clear and attainable.

The following quote is a criticism of the focus of a lot of traditional mathematics teaching, leaving the learners unable to functionally apply the mathematics they have learnt: “For most students, skills learned free from context are skills devoid of meaning and utility” (Steen, 2001). If skills are meant to be used in contexts that are completely *unfamiliar* to learners, then these skills too are devoid of meaning and function, if they approach the question with no understanding of the meaning and function. As mentioned earlier, it is worth noting that familiar contexts may be used to support an understanding of mathematical concepts that may be used in a variety of contexts. *Unfamiliar* contexts become easier to work with once learners become proficient in the skills that are needed to solve such problems.

Steen (2001) defines logical thinking (in quantitative literacy) as “understanding an argument”, which could include one’s own “argumentation” (Meaney, 2007) and the ability to regulate their own thinking. Meaney’s study involves looking into various forms of learners’ reasoning. She manages this by investigating several responses to mathematics questions. The responses involved: verbal articulation; gesturing; pen-and-paper response and post-answer interviews. She devised the term ‘argumentation’ to explain how reasoning is achieved through different forms. Meaney’s descriptions of ‘argumentation’ within ML are explained in more detail later in this chapter.

2.2 An Introduction to PISA and its Relevance to the Study

According to PISA (2000), contexts can be made more complex through a progression that goes from personal to local to national contexts: “an individual’s capacity to identify and understand the role that mathematics plays in the world, to make well-founded mathematical judgements and to engage in mathematics in ways that meet the needs of that individual’s current and future life as a constructive, concerned and reflective citizen”. Furthermore, familiarity of contexts may be linked to the concept of “distance” from the context.

In PISA, situations in Mathematical Literacy are defined in the following way: The situations in which mathematics is used has ‘distance’ from students’ experiences with real-life contexts. “The framework identifies five situations: personal, educational, occupational, public and scientific.” “The idea of *familiarity* in ML is loosely based on this, as my experiences of ML teaching suggested that learners’ mathematical judgements are affected by their life-experiences and experiences with the subject.

The ML taxonomy borrows some ideas from PISA: The reasoning aspect of (TL4) relies on a learner’s understanding of the contextualised problem at hand. PISA refers to ‘distance’, which is similar to the idea of familiarity: the further the learner is from the problem, the more unfamiliar it is. There is also evidence of distance progression in the shift between Thinking Level 2 and 3 and the shift from familiar contexts to a range of contexts. The reasoning aspect links better to the argument that the above-mentioned literature supports the idea that context familiarity particularly affects higher order skills (reasoning and analysing).

The problems addressed in this study can be classified as ‘personal/educational’ and ‘public’, when one relates the problems back to PISA. This provides some support for the design of the study (discussed in the Research Design Chapter).

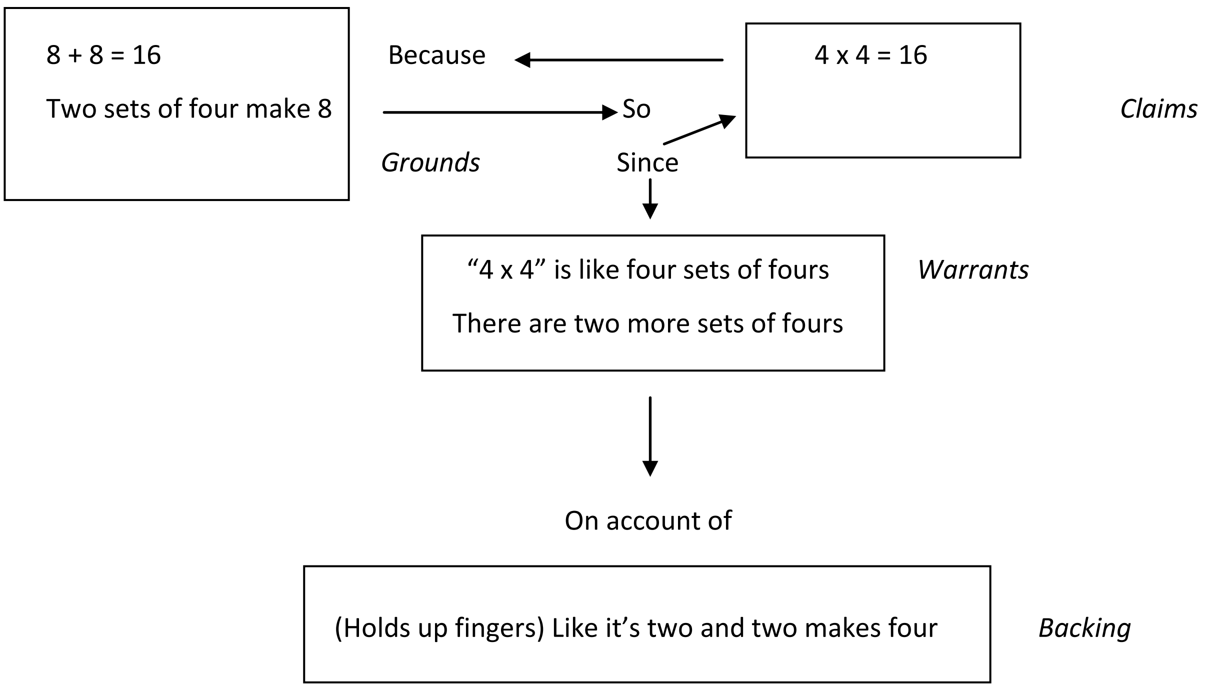
2.3 Introduction to Meaney – Mathematical Literacy and the Effect of Context

In Meaney's study, the relationship between the *familiarity* of context and the appropriate selection of method allows for arguments to be created. The arguments are informed by what is understood in the context for the purpose of solving the problem, which also requires an understanding of the necessary content. Meaney, thus, reflects upon the association between the degree of context *familiarity* (where several versions of the problem was set) and the selection of content needed to solve a problem.

Mathematical Literacy (ML) involves the awareness and the application of selected mathematical content to real-world contexts. According to Meaney (2007), "The context of the task affects what learners perceive to be the most appropriate method to use". This seems to suggest that the selected method varies, depending on the given situation/context.

Meaney's stance is mathematical in focus and she claims that changes in problem context occur in the setup of the problem, which may even refer to phrasing. Pertaining to this study, the focus is on the effect of context familiarity in ML, and Meaney proves to be useful in her explanation of how and when clear arguments would be evident in 'argumentation'. The following illustration shows an example of 'argumentation' that contains evidence that sense has been made of the question/problem.

She claims that the response below shows that a 'convincing argument' has been made using reasoning. She states that this is achieved by making 'clear connections' between the problem and the 'suggested solution'. The model is illustrated on the following page.



The 'connections' that occur between the problem and the suggested solution are words that show that the learners understand that there is a relationship between bodies of knowledge. In this case, we see that the relationships created come in the form of the product above being what it is 'because' of addition. This happens 'since' there are four sets of four. This happens 'on account of' the reasoning that has taken place. The 'argumentation' (Meaney, 2007) shows reasoning through the use of logical connectives that create links between mathematical ideas. This is useful in the study because learners have to show that they have made sense of the problem-context using similar connections.

Meaney's study involves the application of mathematics to practical situations that may take place in real life, but with the focus on being able to work effectively with mathematics on all aspects of life (still mathematical in focus), and the effect that differences in context may have on the way students argue mathematically. Meaney analyses the work that is presented by the students and tries to follow their reasoning when different versions of the same task are given to them. The versions differed as the questions were changed to determine whether responses would be affected. Through this change Meaney made observations and the learners' evidence of work was categorised using the following "Levels of Mathematical Literacy", adapted from Kaiser and Willander (2005):

Level	***Description***
Illiteracy	Ignorance of basic mathematical concepts and methods
Nominal literacy	Minimal understanding of mathematical terms, topics, accompanied by naive theoretical explanations and misconceptions.
Functional literacy	Use of procedures for solving simple problems but these are restricted to very specific contexts and lack in-depth understanding. **Note**: One would have to see the procedure being used in familiar contexts, and not accessed in unfamiliar contexts in order to argue for Functional Literacy.
Conceptual and procedural literacy	Some understanding of the structure and function of central mathematical ideas. **Note**: This category would be for learners who have showed that they can apply a method across both contexts.
Multidimensional literacy	Contextual understanding of mathematics incorporating philosophical, historical and social dimensions

Meaney's idea of argumentation is going to be useful at the overall research model level; argumentation links to the notion of selection, execution and interpretation/sense-making in that it covers the entire process. However, Kaiser and Willander's levels of mathematical literacy may also prove to be handy when providing detail about specific responses.

I acknowledge that the notion of 'distance' may be problematic as a proxy for the notion of familiarity. The mathematical demands within the procedures needed to solve problems also interfere with context familiarity. In the case of this study, the mathematical demands are referred back to the ML taxonomy. This complexity is part of what I hope to investigate through the use of parallel problems incorporating both closed and open-ended questions.

Although Meaney's work relates to mathematical literacy within mathematics, she focuses on mathematical problems that are context-driven. For a better understanding of the purpose of my research, I would like to elaborate and reflect on these aforementioned ideas with ML as the focal point of the research. Meaney's research model has been borrowed in the research design, where learners' methods and arguments across two parallel tasks differing in terms of context familiarity. This will be discussed in more detail in the Research Design Chapter (Chapter 4). Furthermore, the notion of *familiarity* will be explored, as a possible reason for why learners struggle to make sense of some problems in ML derived from real-life situations. According to Meaney (2007), learners struggled to solve problems at a "multidimensional level", which she purported to be potentially linked to contexts that were less accessible to them.

Meaney's study does not explicitly provide examples of learners' responses; she uses the model above to explain how 'logical connectives' exist in learners' argumentation. She presents a table of results that show how learners performed with respect to Kaiser and Willander's levels of mathematical literacy.

The aspect of selection, presented in this study, may be linked to the idea that 'nominal literacy' will suffice, as it involves a basic understanding of the question, which would require them to make the correct selection from the table. The aspect of execution relates mainly to the idea of 'functional literacy', as it involves working with mathematical procedures. The limitations mentioned in that level could be linked to the absence of sense-making.

The idea of 'logical connectives' (Meaney, 2007) and sense-making work well together. Sense-making in this study involves linking the mathematics back to the context, for the problem to be understood completely, while the logical connectives in argumentation show evidence that the learner is linking bodies of knowledge for the entire picture to be understood. In ML this would figure in thinking level 4, where learners are expected to make sense of the problem by relating the mathematics used to the context. The logical connectives would serve to highlight the instances where the learner has made sense.

2.4 Logical Connectives in Argumentation and its Relation to Sense-making

In this study, the analysis of argumentation, as evidenced by coherent ideas reflecting an understanding of the entire problem, will be presented in the analysis chapter, where learners' responses will help to uncover how well they understood the question. This will be done by breaking this process into three interrelated categories:

- ***selection** of relevant data and appropriate mathematical procedures **(A)**;*
- ***execution** of relevant procedures **(B)**;*
- *and reflecting the answer back to the context, for the purpose of **sense-making (C)***.

'Category C' was formed using the idea of Meaney's argumentation. This was done in order to create a category that reveals the way learners link back to the context. The questions designed included open-ended sub-questions because such questions in Meaney's study allow for learners to call upon 'sense-making' skills.

ML is derived from the relationship that mathematics has to real-life situations. Meaney refers to "logical connectives" (Meaney, 2007) that occur within their arguments. "Logical connectives" are words or sentences that clearly show reasoning. The words and sentences show that there is a connection between the context and the mathematics for categories A and C, and within mathematics for category B that the learners have used in order to make sense of the question. This would involve paying attention to words like 'because' and 'if' in learners' argumentation.

Across the three categories (A, B and C), these "logical connectives" are necessary for learners to be able to make complete sense of the problem. In these kinds of open-ended, contextualised problems in ML, for sense to made, the mathematics used needs to be related back to the context using "logical connectives".

When looking at how the levels and the "logical connectives" were defined in Meaney (2007), relevance to this study came to mind. She states that students with a "more diverse understanding of mathematics, with links to other aspects of knowledge" display the highest level of mathematical literacy. A diverse understanding of mathematics involves the ability to apply a method correctly, with an understanding of why and how it is used. Moreover, some learners with this level of understanding are able to apply more than one method, with an understanding of how the different methods relate to one another. Meaney, therefore, uses the term 'logical connectives' in the intra-mathematical frame. In this study, the idea of 'logical connectives' is being applied to learners being able to make appropriate selections from the problem context, execute appropriate procedures accurately and then when learners relate their understanding of the mathematics back to the context.

Meaney's study does not explicitly provide examples of learners' responses, but she uses a model to explain concepts. The exemplification of concepts in her study comes from snippets of responses taken from interviews and work done.

2.5 Meaney's idea of "argumentation" and its relevance to the study

These abovementioned categories were created because of the process that one would have to undergo in order to make sense of an open-ended contextualised problem. The following explains where these categories emerge from:

A) *The **selection** of relevant data is taken from the idea that in order for learners to make sense of the problem, they need to be able to select relevant data and understand what is required from the question. This would be the first step to achieving understanding of the entire problem.*

B) *The selection of appropriate mathematical procedures and the correct **execution** of the chosen mathematical procedures, and respective data, comes after selection, and this aspect will be assessed irrespective of selection. However, if selection and execution is correct, then sense-making becomes easier.*

C) ***<u>Referring the mathematics back to the context</u>*** *becomes the final step, as it requires A) and B) to work together, and then to be followed by an interpretation step. This becomes an exercise of relating the mathematics back to the context.*

Meaney explores how “argumentation” is affected by differences in the “problem context” (Meaney, 2007). She does this by analysing three learners’ responses on a task on measurement. The questions touched on different contexts, and the responses were analysed using Kaiser and Willander’s levels of mathematical literacy (in Meaney, 2007). This literature was chosen to support the notion that *familiarity* of context may be a variable that affects learners’ responses. Kaiser and Willander’s levels will also be used in the analysis chapter to create a rich conclusion that contains more than one way of organising and categorising data.

Meaney (2007) mentions how a learner’s “argumentation” reflects the extent to which the *familiarity of context* has affected their approach towards solving the problem. She refers to the term “argumentation” to describe the reasoning behind the evidence of work presented by the learners. Meaney shows how different versions of the same task affect the way that learners reason. Being able to answer an open-ended mathematical literacy question would involve a “deep and diverse understanding” (Meaney, 2007) of the mathematical content needed to understand a contextualised problem. Thus, sense-making of the entire problem-context would need to become the focus, and the mathematical content is used as a tool to get to that stage. This is similar to Steen’s “well-founded judgment”.

For learners to be able to tackle problems at a “multidimensional level”, they have to show “contextual understandings of mathematics incorporating philosophical, historical, and social dimensions”. Moreover, their “argumentation” would have to express their understanding of the problem, which would also have to include how much sense they made of the context. If learners understand a contextualised problem, their “argumentation” presents evidence of coherent ideas that reflect an understanding of the entire problem, through the correct selection of information and procedures that help learners to make sense of the problem.

In both tests that I am using, I have included a question that is open-ended. It will test the learners' ability carry out multi-step procedures. Thereafter, learners will be required to develop a conclusion that is based on their findings. Thus, requiring learners to make sense of the problem context; this would be evident in their "argumentation".

Meaney has used Kaiser and Willander's levels of mathematical literacy (2005) to show how well learners were able to reason, when moving from one context to another. Contexts that appeared more *familiar* to learners were approached with more confidence, which led to richer, more detailed responses. This study investigates the extent to which *familiarity* of context affects the three categories created. Inspired by Meaney's levels of mathematical literacy, drawn from Kaiser and Willander's levels of mathematical literacy, I developed a grounded identification of levels for each of my three categories. Doing this separately for each of the three categories was useful for this study because:

- their responses in this study may involve a lack in understanding of basic mathematical concepts and procedures. Thus, making the response contain a literacy orientation.

- their responses may involve a mathematical orientation that is plagued with errors and misconceptions.

- their responses may show evidence that there is a basic understanding of the mathematics involved, however they are not able to show an understanding of the context at hand. It can be noted that TL 2 has mainly been used, as knowledge of routine mathematical procedures is the main part of the argumentation.

- their responses contain multistep procedures (TL 3) that are more evident in one of the two contexts (perhaps the one that is more familiar to them). This is then used as the basis for their reasoning when referring back to the context.

- Their responses show competence across both contexts. Both contexts show evidence that multidimensional mathematical steps have enhanced their understanding of the contexts.

The detail of my level descriptors is presented in Chapter 5.

In this study, the learners' "argumentations" will show if context *familiarity* played a role in their ability to work effectively with the three formed categories, used to support their thinking – a point that has already has a significant literature base within mathematics education "It has been known for some time that context does make a difference to the mathematical understanding that is brought to bear" (Meaney, 2007). This will be explored at length in the analysis chapter, as a reflection on the relationship between *familiarity* of context and "argumentation" will be formed in light of the findings. It will be useful to include all the aforementioned frameworks to help assess and analyse learners' responses. The three categories of ML problem-solving used in this study are also closely linked to the theoretical framework used in this study – the mathematisation model developed within Realistic Mathematics Education in the Netherlands (Freudenthal, 1977).

Chapter 3

Theoretical Framework

3.1 Introduction to Realistic Mathematics Education (RME)

As stated in the last chapter, RME is a research based model of maths teaching, originally launched in the Netherlands. The present form of RME is based primarily on several RME writers, but I have chosen to focus on Freudenthal's (1977) view about the nature of mathematics. Freudenthal argued that mathematical procedures could be accessed through the use of 'experientially real' contexts that enable learners to imagine the scenario. This provides better opportunities for learners to access the mathematics that is needed. The term 'realistic' came into being because of the need for mathematics to be related to real-life or realistic situations in order for the mathematics to become more accessible to learners. The contexts chosen related closely to their everyday life or to an 'imaginable' situation, making problems seem *familiar* to them. Thus, RME was started for the purpose of creating problems that learners could imagine, in order to support the process of problem-solving.

The purpose of this study is to focus on the context and to observe the extent to which it promotes a holistic understanding of the problem – with 'holistic' referring across categories A, B and C of the problem-solving process introduced above. My study looks at whether the *familiarity* of context affects the way learners understand the problem as a whole. As RME notes, learners should be able to relate to the problem through their experiences, which opens access to the mathematics needed by allowing them to make sense of the context. I have applied this idea to my hypothesis, which explores (in the context of ML) the ways in which being *familiar* with a context might relate to the way learners: access the mathematics needed to solve the problem; execute the required mathematics; and relate the mathematics to the context for the purpose of sense-making.

One of the main ideas central to RME is of mathematics considered as a human activity; it can never be considered fixed, as human activity evolves (Freudenthal, 1977). In practical terms, the focus is on the growth of the student's knowledge and understanding of mathematics. Therefore, it is focussed on students' progress in mathematics. In the process of this progress, models that originate from contextualised situations are created. These contexts function as bridges to higher levels of mathematical thinking and understanding (Treffers, 1991).

There are two ideas that have been presented above: the first is that mathematics is a human activity; the second is that contexts requiring mathematical models for sense to be made of them are used to develop progressively more complex mathematisation. The second point relates to the first through how human activity involves making sense of situations. The notion of mathematisation brings in both of the previously mentioned points. Freudenthal (1991) describes mathematisation as having two components – horizontal and vertical mathematisation: horizontal mathematisation involves going from the world of life into the world of symbols, while vertical mathematisation means moving within the world of symbols.

There are important similarities between ML and some aspects of RME, which suggested that aspects of RME theory would be useful for my study: in particular, both ML and RME work from the assumption of the necessity of starting in context. However there are also important differences. In RME, the context is viewed as a starting point that supports mathematical sense-making, while in ML the context is the focus and the mathematics serves to make sense of the situation. Although both RME and ML depend on the context for sense-making to occur, the purpose of using context differs considerably. RME requires learners to make sense of the mathematics using the context as a means. ML however requires for learners to make sense of the context using the relevant mathematics as a means.

3.2 The relevance of RME to this study

ML overlaps with RME, as the subject requires learners to make sense of a situation/context using appropriate mathematical methods. RME is focussed on creating “experientially real” (Barnes, 2004) problems that promote an understanding of the relevant mathematical content using contextual support. “Experientially real” problems are designed to relate to learners’ experiences (can be ‘imagined situations’ in RME), in order to act as an aid enabling them to deal with the mathematical demands of the problem. ML-type problems differ from RME type problems because the focus on ML is not mathematical; the focus is on the context and understanding it using the relevant mathematics.

RME is relevant to this study because the notion of ‘experientially real’ contexts can be linked to the idea of context familiarity, as a potential aid, that may assist learners in being able to work with the mathematics required to unpack the context. ML focuses on the understanding of contexts through relevant mathematical means. The process of mathematisation in RME is relevant to this study and will be used as a theoretical tool, facilitating the analysis of learners’ responses. Experientially real problems in RME are designed to enable learners to access the necessary mathematics. The hypothesis is that these contexts/situations provide cues, as they relate to learners’ experiences. In this study, the context becomes the variable that could possibly hinder or support learners’ abilities to select the necessary data and mathematics that would support higher order thinking, which is needed when referring back to the context.

The idea in RME is to create an experience involving a context that learners can relate to in order to support the mathematics that is needed. The underlying premise here is that this idea of contexts learners are able to relate to supports learners in developing sense-making and problem-solving. This idea will have to be reversed in order to achieve the purpose of this study, which requires evidence of whether or not context familiarity affects the way mathematics is selected and executed. Because ML is context-driven, learners are required to understand every aspect of the situation by using the necessary mathematics. Therefore, the study is focussed on context *familiarity*, and its influence on the way learners make decisions about what steps to take in solving problems – essentially on how learners mathematise.

“Experientially real” problems are designed to align with learners’ experiences in order to support them to access the necessary mathematics. Once the necessary mathematics is selected, through their own understanding of what is required mathematically, learners shift to within the realm of mathematics, as RME is essentially mathematical in focus. An experientially real problem, therefore, enables learners to mathematise horizontally and vertically. It creates support for the mathematics needed by allowing learners to make sense of the mathematical procedures needed to solve the problem. This may very well be the case in ML, as the context is often used to make sense of the mathematics. Once the mathematics makes sense to the learner, it is used as a vehicle of meaning that provides insight into other aspects of the context. This in fact would mean that the learner would have to understand what such a problem requires mathematically and in terms of the context.

3.3 Horizontal and Vertical Mathematisation within ML

Referring to “experientially real” problems (Barnes, 2004), this study is focused on investigating whether, and if so, how, learners mathematise when they can relate to (familiar) and when they can less easily relate to (less familiar) context. This is leaning towards the idea that learners are able to relate to the problem context, if they “mathematise” (Barnes, 2004) horizontally and vertically with efficiency. “Horizontal mathematisation” (Barnes, 2004) refers to the ability to relate the context to the mathematics, for the purpose of selecting the necessary mathematics. “Vertical mathematisation” (Barnes, 2004) refers to proficiency within the mathematics; moreover it is the ability to execute the mathematical procedures correctly.

The purpose of ML, in terms of policy, is to enhance the understanding of mathematical concepts through engaging learners in real-life contexts. In order to create a foundation for this study, RME will be explored in terms of “horizontal mathematisation” and “vertical mathematisation” (Barnes, 2004), which captures how learners relate mathematical concepts and skills to their own understanding of the context. The idea in RME is to become proficient in mathematics through contextual aids.

According to RME, learners have the opportunity to reinvent mathematical insights and procedures by applying them to different contexts. This is formally known as “horizontal mathematisation” (Barnes, 2004), which in terms of this study, would involve how learners select relevant information and strategies from the context, through the “judgements” (Meaney, 2007) that they have made. These judgements may be informed by context *familiarity*, which will be present in their work. “Horizontal mathematisation” may be viewed as a form of “argumentation” (Meaney, 2007), as it too is an expression, written or verbal, of what learners are thinking.

Using the notions of “argumentation” (Meaney, 2007) introduced in Chapter 2, within the concept of RME (Barnes, 2004), a basis for the analysis is created. This will, therefore, be useful in exploring how learners access informal strategies to make sense of a particular situation brought by the context. By analysing these strategies, one will be able to see how learners have mathematised horizontally.

Barnes (2004) presents the RME based view of learning thus: “Learners should therefore learn mathematics by mathematising subject matter from real contexts and their own mathematical activity”. This is what distinguishes RME from more traditional forms of mathematics education. By analysing learners’ mathematical activity in light of this idea of mathematics, this study serves to better understand how learners mathematise from the context, in instances where the context selected varies in *familiarity*. With reference to vertical mathematisation, this will be evaluated in terms of how they worked within the realm of the mathematics in both given contexts. Thus, I will be investigating whether the intra-mathematical steps are affected in any way by contextual differences.

In Category B, the focus is on learners' selecting the relevant mathematics and conducting the procedures effectively, the bulk of the work lies. This is because it involves horizontal mathematisation in the form of selecting the mathematics that is needed to solve the problem. Vertical mathematisation comes in when the learners work intra-mathematically, and conduct the procedures effectively.

Category C, which involves mathematical modelling, is not directly covered by RME notions of mathematisation as they stand. This is in line with RME's focus on progressive mathematisation, but leaves untheorised the ML focus on interpretation of answers in context. In order to deal with this sense-making in context idea that is central to ML, I had to adapt the theory – as presented in the following section.

3.4 Reverse Horizontal Mathematisation

Horizontal mathematisation is concerned with going from the context to mathematics, for the purpose of working within the mathematics. In this study, and aligned with the goals of ML, the learners were required to do this in 'reverse' as well as 'forward' directions. The reverse effect is needed for sense to be made in ML, with questions that require learners to reason by relating the answers from vertical mathematisation back to the context.

ML focuses on improving learners' understanding of real-life issues by making them more mathematically literate. This is achieved by engaging them in problems that are derived from real-life experience. This rhetoric is the reason why some higher order questions allow for learners to relate their understanding of the mathematics to the context at hand. This is necessary for learners to understand the problem context through the required mathematics.

ML's focus is to explore contexts that evoke real-life issues. These are solved by using selected mathematical methods. RME often uses contexts that are derived from real-life, although the focus is not to enhance the understanding of the context. RME allows for the mathematics to make sense to the learner using the context. "RME is promising to enhance learners' understanding in mathematics" (Armanto, in Barnes 2004). The point is that you would need to be able to analyse and interpret the use of the mathematics and the answers deriving from this use in order to understand the context.

3.5 Summary of Mathematisation and Sense-Making with Relevance to the Study

- Evidence of "Horizontal Mathematisation" (Barnes, 2004) – The context (the variable of familiarity thereof) may present opportunities for learners to select the relevant data and mathematics needed to solve the problem.

Horizontal mathematisation (HM) occurs when learners are aided by the context to select the mathematics needed. The focus in RME is mathematical, however aspects of RME may be used to define concepts in this study that focus on ML.

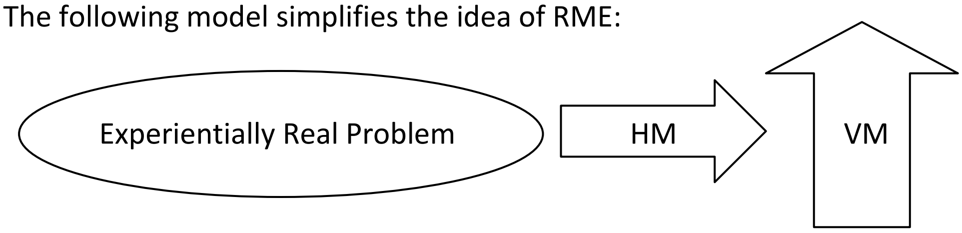
- Evidence of "Vertical Mathematisation" (Barnes, 2004) – The correct execution of the mathematics needed to solve the entire problem.

Vertical mathematisation (VM) occurs when learners work within the mathematical realm of the contextualised problem, i.e. the mathematical procedures that are needed to solve the problem, which is intended for a mathematics problem; however the idea has been adapted to include problems in ML.

- Evidence of reverse HM – When the mathematics used is related back to the context.

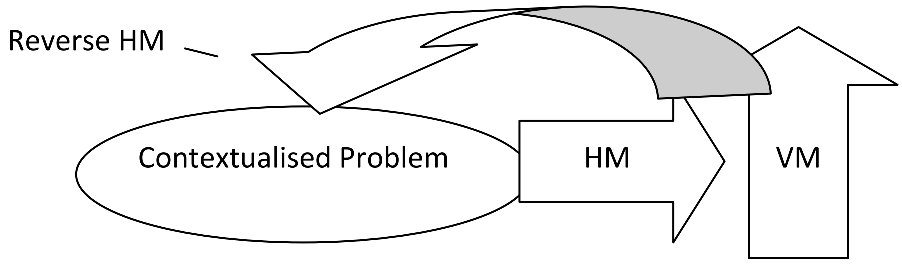
This means that the mathematics is used as an aid to justify thinking in relation back to the context, contrasting HM, which involves the idea that context aids the mathematics needed. Reasoning in open-ended contextualised problems, where ML is the focus, involves the idea that HM and VM are needed to bring the problem-context to light of the problem.

The following model simplifies the idea of RME:



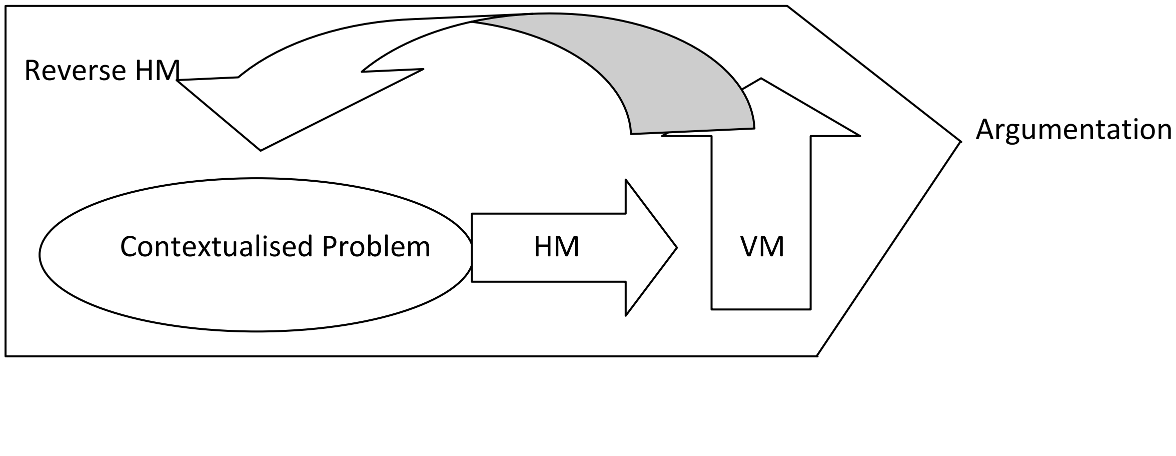
The ‘experientially real’ problem enables the learners to access the relevant mathematics through HM, and then the learner is expected to work within the mathematics by conducting the procedures effectively, which is considered as VM.

The following model simplifies the idea of RME in terms of ML:



The diagram above demonstrates how ML requires for the problem-context to be addressed using the solutions obtained mathematically. Therefore the same applies as the RME diagram, however ML considers that HM in reverse needs to take place in order for the context to be understood through mathematical insight.

The following model simplifies the idea of RME in terms of ML for the purpose of this study, and links it to the idea of argumentation presented earlier:



Category C involves mathematical modelling and tends to be somewhat backgrounded in these RME notions of mathematisation as they stand. This is in line with these authors' focus on progressive mathematisation, but tends to pay less attention to the ML focus on interpretation of answers in context.

For the purpose of the study, the entire diagram shows 'argumentation' (Meaney, 2007), and would need to contain examples of 'logical connectives' between the context and selection, execution and interpretation of both information and mathematical tools.

The study will also explore how the familiarity of context plays a role in the way learners translate a problem into mathematical language. The steps taken and the reason behind their choice of method will benefit the study the most, as it will bring to light whether or not *familiarity* of context is an aspect that is worth scrutinising further.

3.6 Argumentation and the Idea of Realistic Mathematics Education (RME)

Referring to the diagram above, argumentation occurs for both types of mathematisation and for mathematisation in reverse/sense-making. Even in the absence of HM or VM, argumentation occurs when there is an attempt to create an argument, which is an attempt to make sense of the problem.

"Argumentation" (Meaney, 2007) is the logical process through which solutions are created when learners are trying to understand the "problem context". The ideas are presented as evidence of reasoning in instances where learners are able to use higher order thinking to make sense of open-ended problems. In the case of this study, the problems require for learners to present "argumentation". According to Steen (2001), mathematics in context implies that "mathematical tools are used in specific settings, where the context provides meaning". "Horizontal mathematisation" (Barnes, 2004) is based on this notion, where context may be used to make sense of the mathematics needed to solve a problem, which is mathematical in focus. In ML the relationship works in the same way and in reverse, i.e. that the mathematics is be used to make sense of the context.

“Horizontal mathematisation” (Barnes, 2004) in reverse may be equated with mathematical modelling (Gravemeijer, 1997). Modelling, in the context of ML, would involve making sense of the context using the relevant mathematical procedures.

Learners who can display a “diverse understanding of mathematics” can be seen as proficient and flexible in their ability to apply the mathematics needed to solve problems in real-life contexts. Learners who can clearly represent logical connectives (such as ‘if’ and ‘therefore’) in their argumentation, across the three categories, have shown that they have made sense of the problem.

The answers that the learners produce in the tests will be used to investigate their arguments, where linguistic features in the responses (written) are analysed together with the mathematics used to solve the problem. The linguistic features (logical connectives) will determine whether, indeed, learners have made sense of the problem. Written arguments, as evidence, reflect what learners are thinking throughout the problem-solving process, which provides the basis to argue for or against the notion of *familiarity of context* and its effect on learners’ abilities to solve problems.

The levels used in chapter 5, under each of the categories, are derived from learners’ responses. This is considered as a grounded derivation, which forms the basis for ‘grounded analysis’ (Strauss and Corbin, 1990).

Chapter 4

Research Methodology

4.1 General Methodological Strategy

This research is a case study of the effects that context familiarity, as a variable, may have on learners' abilities to solve problems in mathematical literacy (ML). The questions, introduced in Chapter 1, have been presented once again below in order to provide background to the parameters of this section.

4.2 Research Question

In what way does the context affect how learners approach and deal with two ML problems, which are designed to differ on the context familiarity variable?

Sub Questions

1. How do more/less familiar contexts impact on the way learners engage with:

    a) the selection of appropriate information/mathematical tools;
    b) the execution of mathematical procedures and
    c) sense-making of answers in relation to the context.

2. What do the findings on Question 1 tell us about the impact of contextual familiarity on the integrated application of content-based and contextual understandings?

Meaney's idea of 'argumentation', discussed in Ch 2 and 3, proved to be useful when helping to organise and distinguish across a range of responses, as 'argumentation' could be seen in responses across all stages of the contextualised problem-solving process. This included the correct selection of data and mathematics; the effective manipulation of the chosen mathematics; and sense-making. The responses may not necessarily contain evidence of both types of 'mathematisation': learners may attempt to make sense of the problem without containing evidence of 'HM' or 'VM' in their 'argumentation'.

4.3 The data that is going to be collected:

The data is going to be collected for the purpose of determining whether context *familiarity* is a variable worth considering; whether it affects the way learners approach problems in ML. The research strategy is to use two tasks one involving a context that is more familiar to learners and another that is less familiar.

The data comes in the form of two sets of answers from the two tasks that were given. The tasks were designed to be parallel in terms of content, however the contexts differ. The reason for the variations in context is owing to the fact that *context familiarity* is the focal variable.

The research subjects were grade 10 learners from the school where I teach. They wrote the tests at school during class time, and enough time was allocated to ensure that learners responded to the test completely, and to the best of their abilities. I decided to focus on grade 10 learners because they have had limited exposure to ML and therefore, limited exposure to ML-related contextual tasks. This was necessary, as the study is focused on the effect of context familiarity with regards to learners' performances in problem-solving, and therefore, more limited experience with a range of contextual problems was desirable for this study. The content was not the primary focus, but the content and processes needed to execute the procedures across both versions of the task had been covered previously in their GET mathematics learning.

The subjects were broken up into two groups because I decided to incorporate the aspect of 'order' into the study in order to see whether doing the familiar/unfamiliar task first appeared to impact on learners' working with the 'other' task. Group 1 was given the familiar context first, while Group 2 was given the unfamiliar context first. This allowed me to investigate whether or not the order of doing specific tests affected the way learners answered the second test. In this case, order would be an additional variable that I could explore; however it was not the central focus of this study.

The tests formed the instrument used for data collection, as the ‘argumentation’ (Meaney, 2007) presented in learner responses to the two tasks provided sufficient data to determine whether or not *familiarity* affected the way learners solve problems in ML.

Learners’ tests were collected after they completed them, and then the analysis of their argumentation took place. Within their ‘argumentation’, the aspects identified through the theoretical framework linked with the nature of ML in the previous chapters were analysed:

Category A – Their ability to select the relevant data from the given table.

Category B – Their ability to select suitable procedures and execute them effectively.

Category C – Their ability to reflect back to the context, with the mathematical solution in mind.

4.4 Design of Research Instruments

On the next two pages, the more familiar task is presented first, followed by the unfamiliar task. Each task contains a table of data detailing a summary of the number of merits and demerits attained by the learners involved in the study (familiar context), and the number of schools, students and teachers per district (unfamiliar task), which is followed by questions related to the table. All names in the table below are pseudonyms and these pseudonyms were used in the report.

<u>Familiar Task:</u>

<u>Grade 10 Mathematical Literacy Test</u>

<u>Table:</u> Number of merits and demerits earned by 16 Grade 10 learners.

Learner	Male/Female	Age	Number of merits	Number of demerits
Debbie Mercer	F	16	9	8
Michaela Mourao	F	16	16	1
Jessie Charles	F	16	12	8
Stefano Ragelli	M	15	4	4
Jason Roberts	M	15	7	5
Ricky Jordan	M	16	25	1
Jennifer Patterson	F	17	25	5
Nicky Hale	F	16	18	2
Katherine Benson	F	16	13	8
Moipone Sithole	F	15	25	0
Suhail Bismillah	M	16	9	4
Robert Mendes	M	16	12	4
Natalia Nunes	F	17	10	6
Natasha Nunes	F	17	8	5
Kanu Garcia	M	16	1	0
Darius Dlamini	M	16	5	6

The Merit/Demerit system is used at our school as a discipline strategy.

- If learners get **10 Merits**, they are allowed to wear "civvies" on the following Friday.
- If learners get **5 Demerits**, they have detention from 14:30-17:00 on the following Friday.

<u>Refer to the table above to answer the following questions</u>:

1.1 Find the ratio of boys to girls.

1.2 Find the percentage of learners who require one more demerit to get detention.

1.3 Argue for or against the following statement, using the data that is given (refer to table 1 above): "Older students are better behaved". Evidence using calculations and/or graphs must be used to support the points that are made.

Unfamiliar Task:

Grade 10

Mathematical Literacy Test

Table: Number of schools, learners and educators in districts of Free State, 2001.

In order to effectively support districts, the government is considering altering district boundaries to cap the number of learners per district to a maximum number of 65 000.

District	Northern or Southern Free State	Number of schools	Number of educators	Number of learners
Reitz	N	279	1 500	48 700
Phuthaditjhaba	N	67	1 500	50 100
Harrismith	N	195	1 600	51 200
Kroonstad	N	286	2 100	58 000
Odendaalsrus	N	269	1 900	58 000
Sasolburg	N	214	1 900	61 000
Bethlehem	N	244	2 100	61 800
Bloemfontein East	S	108	1 900	62 200
Bloemfontein South	S	172	2 000	63 000
Ladybrand	S	288	2 100	63 100
Bloemfontein West	S	222	2 200	68 400
Welkom	N	115	2 300	70 500
Total	------	2459	23 100	716 000

Refer to the table above to answer the following questions:

2.1 What is the ratio of Northern Free State districts to Southern Free State districts? Write in simplest form.

2.2 What percentage of districts shown above need boundaries changed to satisfy the government max cap figure of 65 000?

2.3 "The more schools in a district, the more contact with teachers your child will get". On the basis of the data given above, what are the arguments for or against this statement? Provide evidence using calculations and/or other graphs to support the points you make.

Locating the task in a school context ensured familiarity to all learners. At one point or another, all the learners have either received merits or demerits for particular reasons and all are aware of the sanctions and structure within which merits and demerits are located. The second task was designed to ensure the content/process is parallel, but less familiar.

The tasks are parallel in the following way: they both contain questions that require the selection of the same mathematical procedures. The first question requires learners to know how to write the form of a ratio. The second question requires that learners are able to express the part of a whole as a percentage. The third question requires broadly the same mathematical procedures to argue for or against a given statement.

Using the taxonomy levels, the questions from both tasks can be analysed. The first question is thinking level one, as it involves the knowledge of concepts, which, in this case, come in the form of knowing how to represent the answer as a ratio. The second question is thinking level two, as it requires learners to conduct basic, routine procedures – writing a fraction as a percentage. The last questions combines both thinking level 3 and 4 in the following way: the third thinking level comes in because there is a need for complex, multi-step procedures to be conducted; the fourth thinking level comes in when learners have to reason and argue 'for or against' the statement. All of the questions require selections of appropriate data and appropriate procedures to solve, or understand, the problem.

It was difficult to create tasks that are exactly parallel. The two tasks differ in the following ways : The familiar task had 16 learners, while the unfamiliar task had 12 districts; the familiar task contained labels for boys and girls (B & G), while the unfamiliar task contained labels for north and south (N & S); the familiar task uses simpler, more accessible language, while the unfamiliar task uses more advanced terms e.g. "...altering district boundaries to cap the number of learners..."; the language demand of question 3 in the unfamiliar task is considerably higher than that of the familiar task. Therefore, unfamiliar context is likely to mean that unfamiliar language is used. The problem is that unfamiliar language can make it harder for learners to access the necessary mathematics separately from context familiarity as a variable. It is also worth mentioning that the size of the numbers were larger in version 2, may have acted as a deterrent to some learners who may have already had problems relating to the context.

One of the factors that remain parallel for both questions is the aspect of scaffolding. Both contexts contain questions that have not been scaffolded. The three questions do not relate to the former or the next. This means that learners have three independent questions to answer. So the answers become three options for useful data to be taken from, rather than wrong answers that affect how learners attempt the next question. 'Not scaffolding helps to maintain higher levels of cognitive demand' (Venkat et al, 2009). Overall therefore, the cognitive demand of the questions from both contexts is similar if not parallel. The questions from both contexts require learners to use the same mathematical methods to support their thinking. As mentioned throughout this section, the familiarity of context has been changed intentionally.

4.5 The Division of Subjects/Learners

The learners were divided into two groups systematically to ensure that each of these groups have similar averages and ranges in terms of prior ML attainment. This is an attempt to make certain that learners have been evenly spread in terms of prior attainment in ML, so that this could be eliminated as a factor that affected the analysis of argumentation. The end-of-term results from the first two terms of 2009 were used to get these mean results. The reason why the first two terms have been used only is because grade 9 mathematics results are not appropriate for the purpose of this study, as the focus of this study is on learners' performances in mathematical literacy. All learners in grade 9 mathematics performed relatively poorly; the maximum mark achieved by the top learner was 67%. Each group consisted of 8 learners.

One group wrote the test with the familiar context first, while the other group wrote the same test second. This was done to determine whether or not order had anything to do with the way learners answered questions. Thus, I could explore whether the first task (familiar or unfamiliar depending on the group) aided learners in being able to do the second task with more proficiency, it should be picked up in the results.

The groups are labelled as follows within the analysis:

Group 1: *Learners/subjects who wrote test 1 (familiar context) first.*
Group 2: *Learners/subjects who wrote test 2 (unfamiliar context) first.*

Comparing the means and ranges of prior performance of the two groups suggested that they were very similar:

Mean/Average of Groups' Results in ML for Terms 1 and 2

Group 1: 57,5%
Group 2: 57,4%

Range of Groups' Results in ML for:

Term 1
Group 1: 35%
Group 2: 37%

Term 2
Group 1: 39%
Group 2: 35%

4.6 Aspect of Order

Group one wrote the familiar context first, while the second group wrote the unfamiliar task first. This is being explored because of the possibility that answering the questions to a task may assist or hinder the learners' ability to answer the next task.

It was expected that learners would provide interesting and in-depth responses to question 3. Due to the complexity of the question, there will be many versions of correct, partially correct and incorrect responses. These responses were analysed with the focus on familiarity, while respecting the idea that order may be a factor that also affected the way learners approached the questions in this study.

4.7 Task Analysis

Both tasks require learners to: select the necessary information from the tables; work with the selected data and mathematics to ensure that the procedures have been conducted correctly; make sense of the problem associated with the given context. The range of cognitive demand varies and progresses, moving from one question to another, which is mentioned in chapter 2. Having more open-ended questions included provides more opening for me to see the different aspects of the mathematisation cycle based on literature discussed in chapters 2 and 3.

4.8 Limitations of the Design

The limitations of the design may have occurred in the design of the task. The tasks are meant to be parallel in terms of the content. Creating parallel tasks involve designing questions that require learners to conduct the same mathematical procedures; however the difference, in this case, is in the contexts that the questions are based on.

It was a challenge to formulate questions that are parallel. The first question was somewhat easier to design, as it has a low mathematical demand (containing a low thinking level). The question is more straight-forward and easier to adapt to the two different contexts. Thus, this question, in both contexts, is more parallel than the others.

The second question was slightly more challenging to design for both contexts, as there was an aspect of language to consider: the language demand of the second question in the unfamiliar task was higher than the one in the familiar task. Phrases and words such as ‘government max cap figure’ and ‘boundaries’ contribute to the increased language demand of question 2 in the unfamiliar contexts. This role of language in mathematical learning has been emphasized in the literature: “Mathematics education begins and proceeds in language, it advances and stumbles because of language, and its outcomes are often assessed in language” (Durkin and Shire, 1991:3).

The third question was certainly the most difficult to make as parallel as possible. This was due to a combination of elements, i.e. the language demand of the unfamiliar context, contained in the question too, and the cognitive demand/thinking level of the question. If one does not know any better, they may think that the questions are completely different. The familiar and unfamiliar tasks are parallel because they require learners to call upon the same mathematical skills that are needed to support arguments.

There is the possibility that learners may be more familiar with certain labels. This may facilitate the identification and selection of the appropriate labels in order to represent the answer as a correct ratio. This may also be compounded by the fact that the familiar task uses language that learners are familiar with. The language in the familiar task is also more accessible for the learners, as it is involves terms that they use often, e.g. detention, merits, demerits, etc.

4.9 Reliability

Reliability refers to the consistency of a measurement tool. In this case one would look into the degree to which the tool measures in the same way if the same subjects are used under similar circumstances. One could also ask if the task/tool has repeatability. The reliability of a tool can only really be estimated and not measured.

The plan has been to test and retest in order to show that the tools are indeed reliable. At two separate times, the instrument was implemented. The two groups had performed similarly across all the levels. Follow-up conversations with the learners after the test was administered suggested that learners had understood test demands and would have performed similarly on a test re-sitting. One has to assume that there is no change in the underlying condition, which may be the trait that is being measured (i.e. familiarity of context).

Under the familiar task, the two groups performed similarly in the two tasks, regardless of order. However, there were three learners from group 1 who were fully competent in all three categories for the familiar context only. They were competent in terms of their ability to present an 'argumentation' that reflected higher order cognitive abilities.

4.10 Validity

Validity would refer to the strength of the conclusions or inferences derived from the study. Cook and Campbell (1979) define validity as the "best available approximation to the truth or falsity of a given inference, proposition or conclusion".

The results across the 8 learners in each group were bound loosely by a pattern that showed that they all struggled more to achieve competency in all three categories, under the unfamiliar context. The only anomaly was presented in the unfamiliar context, where 3 learners were able to make sense of the problem, after selecting the appropriate data from the table, and manipulating the mathematics effectively. This shows that order may have something to do with the way learners approached the familiar context.

4.11 Ethical working

This study involved getting consent from the learners and their parents with regards to using their tasks in the report. The consent forms, attached in the appendix page, have been issued to the entire class. Those who are interested in taking part in the study returned the consent forms. Another consent form was issued to the principal, requesting for school-time to be used to hand out the tasks to the learners.

The learners have been issued with pseudonyms in the report. It serves to protect their identity and it becomes easier to analyse responses, as there is no danger of being offensive, leaving it all for academic purposes.

Chapter 5

Findings and Analysis

5.1 Introduction

In this chapter the data analysis and interpretation using the theoretical framework of RME (Barnes, 2004) linked to Meaney's notion of argumentation as a backdrop, is presented. I began though by using a "grounded analysis" (Strauss and Corbin, 1990). This mode of analysing data is aimed at rooting observations in the data, rather than applying a pre-determined theory.

This method of analysing data was chosen because it is not static and confining. It allows for unexpected and surprising observations to be dealt with alongside those that might have been foreseen within the selection of the research design, core theoretical concepts and also from categories drawn from the literature. This data analysis therefore relies on both grounded and typological approaches. The nature of the research enables for this approach to be used with confidence, as the categories selected were deemed tentative and responsive to potential developments in the process of observation and analysis. In my analysis, I have linked my "grounded" observations to the core theoretical concepts.

The interpretation of findings and analysis are also going to be constructed using the theoretical framework and related literature. Aspects of RME are used in the analysis to assist in determining the extent at which learners have mathematised horizontally and vertically in order to create an argument. This provides evidence of how the learners have made sense of the context.

5.2 Questions Used to Gather Data

<u>Question 1.1</u>: Working with Ratio

The first question asked learners to find a ratio. In the familiar context the learners were asked to find the ratio of boys to girls, while in the unfamiliar context learners were asked to find the ratio of Northern to Southern (Free State) districts. The parallel aspect of the two questions is that they both asked learners to find the ratio from the information given in the table.

Learners were required to read and understand the question in order to select the appropriate information to form the correct ratio. Selection of the correct numbers, represented in an appropriate way, would mean that learners had done the particular question correctly.

Context	**<u>Familiar</u>**	**<u>Unfamiliar</u>**
Question	*Find the ratio of Boys to Girls.*	*What is the ratio of Northern Free State districts to Southern Free State Districts?*
Expected Answer	*7 : 9*	*2 : 1 (or 8 : 4)*
Additional Comments		*Unsimplified option is considered correct as well. This is to ensure that this answer is parallel to the familiar context.*

****

<u>Question 1.2</u>: Working with Percentage

Learners are expected to read and understand this question in order to select the correct proportion/fraction, which needs to be written as a percentage. In the familiar context, learners are expected to understand that learners who require one more demerit to get detention have 4 demerits. They will have to count the number of learners with 4 demerits, and then write it as a fraction out of the total learners, which then gets converted into a percentage.

Context	**<u>Familiar</u>**	**<u>Unfamiliar</u>**
Question	*Find the percentage of learners who require one more demerit to get detention.*	*Find the percentage of districts shown above that need boundaries changed to satisfy the government maximum cap of 65 000?*
Expected Answer	$\frac{3}{16}$ *x 100 = 18.75%*	$\frac{2}{12}$ *x 100 = 16,67%*

****

<u>Question 1.3</u>: Open-Ended Question

This question allowed for a variety of responses, which is the reason why it was created for both contexts. Learners are required to use necessary mathematical skills to make sense of the problem.

Once a mathematical solution has been reached, learners are required to reflect back to the context for the problem to be solved completely.

The open-ended question in the instrument opens up the possibility of seeing all aspects of RME used in the study (HM and VM, and reverse HM), together with the modelling cycle, emerging in learners responses.

Context	<u>Familiar</u>	<u>Unfamiliar</u>
Question	*Argue for or against the following statement, using the data that is given (refer to the table above): "Older students are better behaved". Evidence using calculations and/or graphs must be used to support the points that are made.*	*The more schools in a district, the more contact with teachers your child will get". On the basis of the data given above, what are the arguments for or against this statement? Provide evidence using calculations and/or graphs to support the points you make.*

****

As noted in the Research Design Chapter, the problems from both contexts may be considered "experientially real" in the RME sense (Barnes, 2004), where 'experientially real' focuses on how learners should be able to imagine or visualise the situation, but the second task was certainly less familiar. However, the purpose of this study is to explore whether the *familiarity* of context affects the nature of, and extent to which, appropriate mathematical skills are applied.

I began with various relevant mathematical methods within a "grounded analysis" in this study to draw out and categorise into levels all the different ways that the learners made sense of the problem and context. The different approaches were then located within the RME framework. In this process, I clarified what could be written within the scope of each of the categories, as well – explained later in the chapter. Therefore, RME was a theoretical tool that facilitated the analysis of learners' responses.

5.3 Categories Derived from the Data

I begin this chapter by presenting a summary of the results. Learners' responses were used to form the categories in this chapter linked to features that I have described from both the literature and the theoretical frame. I added level descriptors that were derived in grounded ways, partially because Meaney's study showed levels were useful to distinguish differences between works based on unfamiliar/familiar contexts. However, her categories were unhelpful because they were strictly mathematically orientated, and thus not appropriate for ML. Following this, the categories are going to be explained in terms of how they were derived from the learners' responses; examples of their responses will be included to justify the choice of level under each category, which will be discussed and related back to the literature.

The first category (A) involves the selection of relevant data to solve the problem. The reasons for selecting this aspect as a category are explained below:

Category A – The identification and selection of relevant information

Learners' performances showed evidence of differing selections of relevant information across responses and in relation to context familiarity; this category in this regard proved to be useful. The theoretical framework suggests that learners, when engaging in mathematical activity, mathematise subject matter from "real contexts" (Barnes, 2004), which is specifically known as horizontal mathematisation. This would suggest, specifically in this category, that the selection of relevant data would also form part of this kind of mathematisation. In chapter 3, HM is defined as an ability to move from the context to a mathematical realm, in order to find the mathematics that is suitable for the context. In the data, this aspect was evident in most cases, as the learners attempted to move from the context to the mathematics, for the purpose of understanding the question. HM occurred in all three questions; the degree of cognitive demand progressed as the learners moved from questions 1 to 3.

This category was created because learners needed to make a decision as to what to do with the data provided in the test. The Subject Assessment Guidelines (2005) suggest that tasks need to be "contextually based, requiring learners to select and use mathematical content in order to complete the task". The SAGs incorporate the 'selection' of content, while the study looks into whether students have selected data efficiently. The selection of content in HM involves being able to determine effectively what is required in order to solve the entire problem.

The second category (B) involves the mathematical procedures that have been selected and executed using the data selected. Thus the second category (B) deals with the operations and calculations that have been selected:

Category B – Selecting appropriate procedures/ executing appropriate operations/calculations using the relevant selected data.

This category has been created due to the fact that it takes up most of the learners' responses, as learners feel the need to show their thinking through their mathematical steps. It is worth noting that different levels of execution occur as the cognitive demand increases. In question 3, learners are required to execute multi-step procedures in order to understand the question. Learners in this category are not only expected to select the relevant information, but also the appropriate mathematical tools that will enable them to understand the question. This category would be common in everyday mathematical practice and mathematical literacy. Most learners attempted to perform a calculation, either with the relevant data or with whatever else they thought was necessary for sense of the problem to be made. It is mentioned in the SAGs (2005) that learners should solve problems that are contextually based by using relevant mathematical tools (procedures and operations). A range of content may be drawn upon to solve a single problem, and this must be used by the learner to make sense of "real-life, everyday meaningful problems" (SAGs, 2005). This category was also created from the various attempts that learners made at making sense of the problem through the selection and execution of appropriate mathematical procedures.

Horizontal mathematisation involves the use or selection of "informal strategies" that allow learners to solve the problem, and vertical mathematisation is the part of the problem solving process where “mathematical language and/or algorithms” are being developed. Category B, as I have described it, therefore, contains elements of vertical and horizontal mathematisation, which will be mentioned separately and in detail in the analysis part of this chapter. I found it useful to work with Category B in these terms because there are aspects in the learners HM that involves both the selection of relevant information from the table and the appropriate mathematical procedures. What is also useful to point out in this study is that RME involves context selection for the purpose of supporting learners’ abilities to select particular mathematical procedures. In RME, the focus is mathematical and the contexts serve to support a learner’s understanding of mathematical concepts through the practise. However, in ML the context itself matters, so it was useful to separate the selection of relevant information from the selection of suitable procedures.

The third category (C) involves reasoning by reflecting back to context. This is achieved by referring to the answers derived from calculations and seeking insight on what these answers can be interpreted, or how they speak back to the context. For this reason, the following category was derived:

Category C – Referring back to the context and reflecting on the solution

As suggested within RME (mentioned in chapter 2), the open-ended question has been designed to create a situation involving a context and the opportunity/motivation for the relevant mathematics to be used, but in line with the needs of ML, I also incorporated a question that called for answer to be interpreted in context.

Question three is pitched at different levels, requiring learners to also engage in analysis and reflection, through finding connections between the context and the mathematics needed to solve the problem. The SAGs (2005) refer to the taxonomy as a tool that describes the cognitive demand of a question. This category would be related to level 4 because it involves reasoning and reflecting, which are skills that are necessary for relating the relevant mathematical methods back to the context.

As explained in Chapter 3, in this situation, a learner would be engaging in horizontal mathematisation in reverse. With horizontal mathematisation, learners use the context to select the relevant data and the necessary procedures for the purpose of problem solving. In reverse, learners would be making sense of the problem, where the context is the focus in relation to the answers that have been calculated. Here the mathematical procedures are used to support the argument, which involves trying to solve a problem with respect to the context.

In RME, contexts and related questions endorse an understanding of the relevant mathematical content, which may only be the case if the context relates closely to a learner's experience. This is what 'experientially real' refers to. The context seems to affect what students perceive to be the most relevant approach to use. These are reflected in their argumentation (Meaney, 2007), or written evidence of thinking. According to Meaney (2007), a learner's judgement, which is the reasoning behind their written evidence of thought, is affected by differences in problem context. As mentioned in chapter 2 and 3, Meaney's framework is used to support the notion that learners 'argumentation' is evident when there is an attempt to work with the mathematics in order to justify the question. There were differences that occurred in the learners' responses that allowed for the levels of categories to be formed. These indicators capture the differences that are evident in all the responses that were analysed. Below is a description of the levels within each of the categories, which contains an exemplification of each.

5.4 Description of the Levels of the Categories and Exemplification

Within the above categories, levels were created. These are informed by my analysis of the range and depth of responses. Each response has been slotted into at least one of the categories, which provided a way of determining whether or not the levels were appropriate. An overview of the categories, with all the learners' responses is provided below.

Descriptions of the levels of each category are given, accompanied thereafter by an example of such answers. This is derived from learners' responses, which would contain the best, most detailed examples of each level. For the purpose of clarity, a summary of this aforementioned report has been supplied in the findings. The particulars, brought up in the exemplification, will provide the reader with evidence of different responses. Analyses have been drawn from the findings, which highlight aspects from the research that are crucial to addressing the problems posed in the beginning. This will be presented after the exemplification of levels is shown.

* In this section, the number of levels depended on the range and depth seen in my data.

A – selecting relevant data/information

<u>Level 0</u> – Learner has made no attempt at selecting the relevant data from the given context.

Question 3 (unfamiliar):*"The more schools in a district, the more contact with teachers your child will get". On the basis of the data given above, what are the arguments for or against this statement? Provide evidence using calculations and/or other graphs to support the points you make.*

Response

The more schools in the area, the more education your child will get.

'There are more schools in the Southern Free State, but there are more educators in the Northern Free State.' (Natalia Nunes)

Natalia showed in her response that she was <u>not able to select the relevant data</u> from the table. Her response is written in the form of text; she was not able to select the appropriate mathematics needed to create a more effective argumentation.

****

<u>Level 1</u> – Learner has not selected the relevant data from the given context. The selection might have been random or motivated by a misconception.

Question 2 (unfamiliar):

What percentage of districts shown above need boundaries changed to satisfy the government max cap figure of 65 000?

Response

12 districts:

$$\frac{12}{65000} \times 100$$

= 0, 02% (Darius Dlamini)

Darius selected inappropriate information from the text. This example also shows that a correct procedure has been executed with incorrect numbers.

The answer exemplifies that the selection of appropriate information needed to be separated from the selection of appropriate procedure – This learner shows competence in category B, but not category A.

****

<u>Level 2</u> – Learner has selected the relevant data from the given context.

Question 2 (familiar):

Find the percentage of learners who require one more demerit to get detention.

Response

$$\frac{3}{16}\times 100$$

= 18, 75%

- *3 represents how many people are close to having detention.*
- *16 represents the whole number of learners. (Suhail Bismillah)*

The solution reflects that the learner has selected what data is required in order to solve this question.

****

B – Selection of appropriate procedures, and the correct execution thereof

<u>Level 0</u> – Learner has made no attempt at selecting and conducting any suitable mathematical procedures.

Question 3 (familiar):

Argue for or against the following statement, using the data that is given: "Older students are better behaved". Evidence using calculations and/or graphs must be used to support the points that are made.

Response

I think that older students are more behaved because they have got fewer demerits than the younger students. The younger students have got more demerits compared to the older students, which tells us that old students know how to behave themselves. But younger students work much better than the older students in getting merits. Age 16.

Age 17	*Merits*	*Demerits*
3 learners	*43*	*16*
Age 16	*Merits*	*Demerits*
10 learners	*120*	*42*
Age 15	*Merits*	*Demerits*
3 learners	*38*	*9*

(Darius Dlamini)

Although there is evidence that Darius selected data from the table, there is no evidence that he selected the mathematics needed to understand the problem.

****

<u>Level 1</u> – Learner has made an attempt at conducting suitable mathematical procedures, however the choice of method does not suit the context. This may be due to the lack in understanding of the question.

Question 3 (unfamiliar):

Response

This statement is true; therefore if there are more schools, your children will be in more contact with teachers.

Northern Free State:
<u>Number of Schools</u>
279 + 67 + 195 + 286 + 269 + 214 + 244 + 115
= 1669

<u>Number of Learners</u>
48 700 + 50 100 + 51 200 + 58 000 + 58 000 + 61 000 + 61 800 + 70 500
= 459 300

<u>Number of Educators</u>
1500 + 1500 + 1600 + 2100 + 1900 + 1900 + 2100 + 2300
= 128 300

- *Same was done for Southern Free State*

(Nicky Hale)

Nicky selected the method of adding the entire data for the Southern and Northern Free State under each category. A mathematical method was selected, but it was not an appropriate selection.

****

<u>Level 2</u> – Learner has made an attempt at conducting suitable mathematical procedures. The choice of method is appropriate, but contains errors and/or missing steps throughout their response.

Question 3 (unfamiliar):

Response

The average number of learners per school is 291 (716 000 ÷ 2459). The average number of teachers per school is 9 (23 100 ÷ 2459), if we calculate 291 ÷ 9 = 32 learners for every one teacher.

With the evidence stated above, it is true that the more schools there are in the district the more contact students will have with teachers. However, in some districts there are fewer schools but more students than its previous district (e.g. Phuthaditjhaba to Reitz). It is evident that these districts will not receive as much contact (w/teachers) as others due to the lack of schools and teachers.

(Suhail Bismillah)

Suhail made the correct selection of procedure; however there are steps missing that will ensure that the problem context is completely understood.

Suhail failed to understand the question completely, as there needs to be a comparison between districts in order to prove or disprove the statement.

****

Level 3 – Learner has made an attempt at conducting suitable mathematical procedures. The choice of method suits the context and the method is mathematically sound.

Question 3 (familiar):

Response

The older students are not necessarily better behaved because in some cases the younger students have more merits and less demerits than the older students.

* The rest of the answer continues on the next page.

Average for 17 year olds (merits)

25 + 10 + 8

= 43

∴ 43 ÷ 3

= 14,3333...

= 14

Average for under 17 year olds (merits)

9 + 16 + 12 + 4 + 7 + 25 + 18 + 13 + 25 + 9 + 12 + 1 + 5

= 156

∴ 12 merits per person

Average demerits (17 year)

5 + 6 + 5

= 16

∴ 16 ÷ 3

= 5,3333...

= 5 demerits per person

Average demerits (under 17)

8 + 1 + 8 + 4 + 5 + 1 + 2 + 8 + 0 + 4 + 4 + 0 + 6

= 51

∴ 51 ÷ 13

= 3,92

= 4 demerits per person

(Robert Mendes)

****

C – Relating the mathematical solution back to the context

Level 0 – Learner has made no attempt to relate the mathematical solution back to the context.

Question 3 (familiar):

Response

16 → 9 + 16 + 12 + 25 + 18 + 13 + 9 + 12 + 1 + 5 = 95

15 → 4 + 7 + 25 = 36

17 → 10 + 8 + 25 = 43

(Natalia Nunes)

Natalia has selected the relevant data, and has chosen a procedure that has limited her. She has not related the answer back to the context.

****

<u>Level 1</u> – Learner has made an attempt to relate the mathematical solution back to the context, even though they were not appropriately related.

Question 3 (unfamiliar):

Response

Not necessarily because older students also have more demerits than younger students, in the table there are many older students with more demerits than younger ones, results also vary due to the fact of the different behaviour of students and their work ethic as some learners may acquire merits for academics and schoolwork as well as good behaviour, and the majority of girls have more of the same age group.

Females – 9 : 43 (demerits)

Males – 7 : 60 (demerits)

It seems more girls get more demerits than boys rather than younger students being worse behaved the % being:

9 x 100 ÷ 16 = 56, 2%

(Jessie Charles)

Jessie has chosen mathematics that is not suitable; however the focus is on how the student has related the mathematics back to the context. In Jessie's case, he started with the conclusion, providing an overall impression of the table, before working with the mathematics.

****

Level 2 – Learner has related the mathematical solution back to the context, however there are some inaccuracies.

Question 3(unfamiliar):

Response

It seems that when the number of learners are divided by the number of schools, the average is 291 per school and then to see if contact could be made between children and teachers successfully, divide the number of learners by that of the educators 23 100 ÷ 716 000 = approximately 30 and the educators to schools approximately 23 100 ÷ 2459 = 9,3 amounting to estimates of 291 learners in schools average to 9,3 teachers more schools in a district would be my opinion mean more contact with learners and teachers.

(Suhail Bismillah)

Suhail has got the right idea and understands the factors, in this case, that affect the argument in question. However his argument is flawed because he has not got a full grasp of the mathematics he used. This limited his train of thought and ability to reason, and relate the mathematics back to the context.

****

Level 3 – Learner has related the mathematical solution back to the context accurately.

Question 3 (familiar):

Response

As stated in the data, the results show students of higher ages have more merits than demerits when compared to themselves and more than lower aged students when compared.

Learners over the age of 15 have 163 merits (summed up)
Learners under the age of 15 have 36 merits (summed up)

199 ÷ 16 = 12,4
Therefore, learners with over the average number of merits are over the age of 16, which proves that "older students are better behaved".

(Robert Mendes)

****

5.5 Summary of Categories and Descriptors

	Category A	**Category B**	**Category C**
Level 0	*Learner has made no attempt at selecting the relevant data from the given context.*	*Learner has made no attempt at conducting any suitable mathematical procedures.*	*Learner has made no attempt to relate the mathematical solution back to the context.*
Level 1	*Learner has not selected the relevant data from the given context. The selection might have been random or motivated by a misconception.*	*Learner has made an attempt at conducting suitable mathematical procedures, however the choice of method does not suit the context. This may be due to the lack in understanding of the question.*	*Learner has made an attempt to relate the mathematical solution back to the context, even though they were not related.*
Level 2	*Learner has selected the relevant data from the given context. The selection might be random or motivated by a correct understanding of the question.*	*Learner has made an attempt at conducting suitable mathematical procedures. The choice of method is appropriate, but contains errors and/or missing steps throughout their response.*	*Learner has related the mathematical solution back to the context, however there are some inaccuracies.*
Level 3		*Learner has made an attempt at conducting suitable mathematical procedures. The choice of method suits the context and the method is mathematically sound.*	*Learner has related the mathematical solution back to the context accurately.*

5.6 Findings

The table below consists of the number of correct responses of each question under both groups. The groups consisted of 8 learners each. As a reminder, Group 1 wrote the familiar task first, which was followed shortly by the unfamiliar task, whilst Group 2 wrote the unfamiliar task first etc.

Test Result

KEY:

F – Familiar ***U - Unfamiliar***

Category	Level	Fraction of Responses					
		Question 1		Question 2		Question 3	
		Group 1	Group 2	Group 1	Group 2	Group 1	Group 2
A **The identification and selection of relevant information**	**0**	F - $\frac{0}{8}$ U - $\frac{0}{8}$	F - $\frac{0}{8}$ U - $\frac{0}{8}$	F - $\frac{0}{8}$ U - $\frac{0}{8}$	F - $\frac{0}{8}$ U - $\frac{0}{8}$	F - $\frac{0}{8}$ U - $\frac{1}{8}$	F- $\frac{0}{8}$ U - $\frac{2}{8}$
	1	F - $\frac{1}{8}$ U - $\frac{0}{8}$	F - $\frac{1}{8}$ U - $\frac{0}{8}$	F - $\frac{1}{8}$ U - $\frac{5}{8}$	F - $\frac{1}{8}$ U - $\frac{5}{8}$	F - $\frac{2}{8}$ U - $\frac{5}{8}$	F - $\frac{6}{8}$ U - $\frac{4}{8}$
	2	F - $\frac{7}{8}$ U - $\frac{8}{8}$	F - $\frac{7}{8}$ U - $\frac{8}{8}$	F - $\frac{7}{8}$ U - $\frac{3}{8}$	F - $\frac{7}{8}$ U - $\frac{3}{8}$	F - $\frac{6}{8}$ U - $\frac{1}{8}$	F - $\frac{2}{8}$ U - $\frac{2}{8}$
B **Selecting appropriate procedures/ executing appropriate operations/calculations using the relevant selected data**	**0**			F - $\frac{0}{8}$ U - $\frac{0}{8}$	F - $\frac{0}{8}$ U - $\frac{0}{8}$	F - $\frac{1}{8}$ U - $\frac{1}{8}$	F - $\frac{2}{8}$ U - $\frac{4}{8}$
	1			F - $\frac{0}{8}$ U - $\frac{4}{8}$	F - $\frac{0}{8}$ U - $\frac{2}{8}$	F - $\frac{2}{8}$ U - $\frac{3}{8}$	F - $\frac{3}{8}$ U - $\frac{2}{8}$
	2	F - $\frac{1}{8}$ U - $\frac{0}{8}$	F - $\frac{1}{8}$ U - $\frac{0}{8}$	F - $\frac{0}{8}$ U - $\frac{1}{8}$	F - $\frac{0}{8}$ U - $\frac{0}{8}$	F - $\frac{2}{8}$ U - $\frac{4}{8}$	F - $\frac{3}{8}$ U - $\frac{2}{8}$
	3	F - $\frac{7}{8}$ U - $\frac{8}{8}$	F - $\frac{7}{8}$ U - $\frac{8}{8}$	F - $\frac{8}{8}$ U - $\frac{3}{8}$	F - $\frac{8}{8}$ U - $\frac{6}{8}$	F - $\frac{3}{8}$ U - $\frac{0}{8}$	F - $\frac{0}{8}$ U - $\frac{0}{8}$
C **Relating the mathematical solution back to the context**	**0**					F - $\frac{2}{8}$ U - $\frac{0}{8}$	F - $\frac{1}{8}$ U - $\frac{0}{8}$
	1					F - $\frac{1}{8}$ U - $\frac{5}{8}$	F - $\frac{2}{8}$ U - $\frac{3}{8}$
	2					F - $\frac{5}{8}$ U - $\frac{3}{8}$	F - $\frac{3}{8}$ U - $\frac{3}{8}$

	3					F - $\frac{0}{8}$ U - $\frac{0}{8}$	F - $\frac{2}{8}$ U - $\frac{2}{8}$

5.7 Analysis

In this section, learner's responses will be analysed according to how well they mathematised horizontally and vertically, which in turn contributes to perform horizontal mathematisation in reverse.

The familiar and unfamiliar (u/f) variable is at the centre of this research. This was achieved through the design of questions that were parallel in terms of the mathematical content that could be drawn upon to answer the questions ML. The difference, however, was in the context under which the questions are based. One context has been designed to be more familiar than the other.

The focus in the analysis is to determine if familiarity of context has had an effect on the way learners select and use mathematical content to solve any problem in ML. By breaking down the entire problem-solving into the aforementioned categories, it becomes more manageable and effective to pin-point examples or situations where learners may have been aided by the context.

Based on the categories that were selected, the analysis will be structured according to: whether or not learners selected the relevant information from the tables, the mathematical activity that took place for the purpose of sense-making, and the final step of relating the mathematics back to the context, for sense-making to become complete.

Category A

In questions one and two, the students did similarly well, and no anomalies were identified. However, in the third, open-ended question the students in the first group performed better, being able to select the relevant data from the table. In this case, order played a role, assisting the first group better. The second group was not as able to select appropriate data from the table; this may have been influenced by the fact that the group started with the unfamiliar task first, which potentially could have affected the group's ability to select relevant data from the table.

When referring back at the table, one can see that the majority of the learners were able to work competently in level 2 under category A, specifically in the familiar context, as identified in the table of results above. Therefore, it is evident that learners were better able to select the data from the table that is needed.

The response, shown below, presents an example of level 2, under Category A, being achieved with competence.

A learner from Group 1 gave the following responses for the familiar task:

1.1) *7 : 9*

1.2) $\frac{3}{16}$ *x 100% = 18%*

1.3) *As stated in the data the results show students of higher ages have more merits than demerits when compared to themselves and more than the lower aged students...*

Learners over the age of 15 = 163 (summed up merits)

Learners under the age of 15 = 36 (summed up merits)

(Total) 199 ÷ 16 = 12,4

Therefore, learners with over the average amount of merits as 12 are over the age of 16, which proves "older students are better behaved".

This shows the student has made a correct selection of data taken from the table. This was achieved across all three questions. Not one student managed to achieve this in the unfamiliar task.

Category B

When looking at category B, care needs to be taken when analysing the horizontal and vertical mathematisation that has undergone. The care lies in pointing out instances where they possibly could have been aided by context familiarity, or hindered by the unfamiliarity of context.

Many different methods and approaches emerged from the data on question 3. In terms of the mathematics used, rate and mean were the main methods that surfaced from learners' written evidence.

When referring back to the table, one can see that there is nothing that stands out for the first two levels of Category B. However, it is clear that three learners from group 1 were able to work effectively intra-mathematically for question 3 of the familiar task. This compares to learners being able to give effective intra-mathematical solutions in Gp 2.

The response, shown below, presents an example of level 3 under Category B, being achieved with competence.

A learner from Group 1 gave the following responses:

1.2) $3 \div 16 \times 100 = 18{,}75\%$

$\approx 19\%$

1.3)

Average merits for 17 year olds:

25 + 10 + 8 = 43

43 ÷ 3 = 14, 33

An average of 14 merits per person

Average merits for under 17 year olds:

9 + 16 + 12 + 4 + 7 + 25 + 18 + 13 + 25 + 9 + 12 + 1 + 5 = 156

156 ÷ 13 = 12

An average of 12 merits per person

Category C

Within the third category, learners are expected to reason, reflecting back to the context during the problem-solving process. Understanding the entire process of problem solving involves being able to understand how the mathematics relates to the context. The open-ended question (question 3) has created an opportunity for horizontal and vertical mathematisation to take place. The question therefore provided openings for learners to engage in both the mathematics and the context, relating them to one another, and therefore, to make sense of the entire problem.

Question 3 demands a higher level of thinking, involving a thorough understanding of the context, and mathematics needed to solve the problem, with research findings suggesting that there are more openings for reasoning here, and therefore, more room to see differences in approaches across the two parallel questions. The "familiarity" of context is the variable being explored in this chapter. How this has been achieved is through determining how well learners were able to apply appropriate mathematical skills to two different contexts for the purpose of sense-making. In terms of the methods, "horizontal and vertical mathematisation" are being used as theoretical tools to determine how well the learners have made sense of the questions through mathematical means, if present at all.

The response, shown below, presents an example of a level 3 solution under <u>Category C</u>.

A learner from Group 1 gave the following response:

1.3) The older students are not necessarily better behaved because in some cases the younger students have more merits or less demerits than the older students than the older students.

I selected the following answer because this student was able to refer the correct calculated solution back to the context. Furthermore, this student showed maturity and consideration for all the necessary aspects that need to be mentioned.

5.8 The familiar/unfamiliar variable

The findings that emerged from the data reflect that learners felt more comfortable performing calculations in the familiar context, even though it may have contained errors and misconceptions. In the familiar context there were more examples of partially and fully correct responses across all three questions.

According to the findings (refer to the table above), the following emerged:

- Question 3, under Category A, shows that more learners were able to select the relevant data from the table.

- Under Category B, also for question 3, there is evidence that suggests that only some learners who worked with the familiar context where able to achieve the highest level: Learner has made an attempt at conducting suitable mathematical procedures. The choice of method suits the context and the method is mathematically sound. This level was achieved for question 2 by all the learners in group 1, but only for the familiar context. In the unfamiliar context, the findings show that there is a general inability to select the appropriate mathematics needed to understand the problem.

5.9 General analysis

Question 3 was, therefore, created to force learners to use a higher level of thinking. As per the Subject Assessment Guidelines for ML, a higher level of thinking in ML requires learners to conduct multiple mathematical procedures with the intention to make sense of the problem, relying on reflective thinking and reasoning skills. The focus, in this particular thinking level, is on being able to reason, and reflect on the problem as a whole. Thus, connections within the mathematics, and between the mathematics and the context need to be made in order for the problem to make sense. If learners were more familiar with the context, they were able to mathematise horizontally and vertically with purpose.

The unfamiliar task presented somewhat of a barrier for many, as they did not seem to approach the question with the same ability to apply mathematical skills in order to make sense of the problem. This was different in the familiar context, where more attempts from different learners at solving the problem were apparent. Moreover, more appropriate mathematical tools were selected in the familiar context. Thus, learners found it easier to "mathematise horizontally" (Barnes, 2004) because of the frequent selection of relevant methods, potentially for sense-making to occur. Most learners in the familiar context were also able to execute mathematical procedures with better precision than in the unfamiliar context. They were, therefore, able to mathematise vertically with more confidence and accuracy.

The following example, taken from the familiar context, shows that one learner from a small group was able to operate at this level:

AVERAGE FOR 17 YEAR OLDS:

$$25 + 10 + 8$$

$$= 43$$

$$\therefore 43 \div 3$$

$$= 14.33$$

This translates to 14 merits per person.

This learner has managed to work effectively under Category A and B, showing that she selected appropriate data from the table, which has been used correctly when mathematising vertically.

What leads learners to select the procedures that they do is the idea of “horizontal mathematisation” (Barnes, 2004), which is about informal strategies that may enable learners to make sense of the problem, through understanding the context and the mathematics needed to solve the problem. The learner’s ability to perfect the necessary technique is part of the ability to “mathematise vertically” (Barnes, 2004).

5.10 Summary of findings

5.10.1 Unfamiliar/familiar

Under Category A (selection of data), learners were better able to select information appropriately in the familiar task. With reference to the above table of results, more learners were able to achieve level 2 in the familiar context compared to the unfamiliar.

Under Category B (selection and execution of procedures), learners were better to execute procedures effectively in the familiar task. With reference to the above table of results, three learners in the familiar context were able to achieve level 3, while no learners in the unfamiliar context were able to achieve level 3 in the unfamiliar context.

Under Category C (referring mathematics back to the context), learners were better able to interpret answers in the familiar context than in the unfamiliar context. Learners in the familiar context were better at referring calculated solutions back to the context. With reference to the above table of results, collectively (with reference to level 2 and 3) more learners in the familiar context made better attempts at relating the mathematical answers back to the context.

5.10.2 Order

Under Category A, doing the familiar task first had something to do with the fact that the students in Group 1 were better able at selecting the relevant data from the table. When referring to the table, six out of eight learners from the first group were able to achieve level 2, which involves being able to select appropriately from the table.

Under Category B, doing the familiar task first also affected the way the learners in Group 1 selected and executed the relevant mathematical procedures. From the table of results above, it is clear that more learners in Group 1 in the familiar context were better able at selecting and conducting relevant procedures. Thus, these students achieved level 3, which states that the learner has made an attempt at conducting suitable mathematical procedures; the choice of method suits the context and the method is mathematically sound.

Under Category C, Group 2 managed to achieve the highest level, which states that they were able to relate the mathematical solution back to the context accurately. The issue, however, is based on the fact that the learners related the mathematical solution correctly back to the context, although there are flaws under Category B. Two learners in Category B were able to achieve level 3.

Doing familiar tasks helped in particular on question 3 (more open-ended task). 3 learners who did the familiar task first were able to provide the highest level of response compared to unfamiliar tasks across the first two categories. The third category presented somewhat of an anomaly in the form of the fact that the last level of Category C was achieved by two learners in the second group.

Order in terms of familiar versus unfamiliar first seemed to make little impact on more closed tasks. Questions 1 and 2, in the table of results, show little or no difference between familiar and unfamiliar contexts.

Chapter 6

Conclusions, discussions and reflections

Familiar tasks seem to support the aspects of problem-solving that were the focus in this study: selecting information; selecting and executing appropriate procedures; and interpreting answers in relation to the context. From the evidence presented in Chapter 5, one can note that the findings support the idea that familiar tasks allow for learners to access skills that are necessary for the problem to be solved.

In terms of order, dealing with more open-ended problems first appears to better support how learners deal with both contexts together, provided that the tasks are parallel. Therefore, the order would require learners to go from familiar contexts to less familiar contexts.

6.1 Links to the literature

6.1.1 ML taxonomy, referring to NCS/SAG/CAPS

The ML taxonomy makes reference to thinking level 2 and 3, which differ by moving from working with 'routine procedures' in familiar contexts to working with 'multi-step procedures' in a range of contexts, which would include both familiar and unfamiliar contexts.

Kaiser and Willander would refer to the above-mentioned point as moving from 'functional literacy' to 'conceptual and procedural literacy'. These involve moving from being able to apply basic procedures to applying more complex and varied procedures in less familiar contexts.

6.1.2 PISA

The notion of progression is supported by the data and findings. The idea of progression, according to Pisa (OECD, 2009), refers to going from 'personal' contexts to 'local', then 'national' and 'international', which shows that there is movement in terms of 'distance'. The distance is determined by how the learners can relate to the context. 'Personal' would refer to something that is familiar to them; something that they find easier to relate to.

In relation to the NCS, the application of routine procedures in familiar contexts may relate to the notion of 'personal distance', as we can see that they both involve being able to identify oneself with the problem. As one progresses to less familiar contexts, this may be because the 'distance' from the personal increases.

A dimension brought out in the study is closed questions versus open-ended. This presents an interesting aspect to consider, as closed-ended questions seem not to be affected by familiarity of context and order. The open-ended questions presented interesting anomalies that helped to confirm that familiarity affected the way learners approached the problem. In the findings, it can be taken into account in the findings that order also comes into play in the shift from simple, closed questions to more open-ended ones.

6.2 Implications

According to the study, it will help to progress from familiar to unfamiliar contexts. The idea in the findings shows that the familiar context enables learners to access all three categories, mentioned earlier, better. This enables some learners to access the same categories with similar competency.

It is worth noting that there needs to be progression from closed-ended questions to more open-ended questions. This may prove to be useful, as the intention at policy level is to work with routine procedures in familiar contexts and progress to multi-step procedures in a variety of contexts.

6.3 Reflections

I have learned in the process that one cannot make assumptions on why learners struggle to perform in certain problems. The study has brought to light that there are aspects in ML that affect the way learners acquire skills. I have learned that context familiarity is a tool that is useful for accessing mathematical knowledge and skills.

For my own teaching, I have learned to improve my practise by using more familiar contexts to support access to the necessary mathematics needed for learners to reflect and reason effectively. There is a need in my teaching to make my learners of the progression from familiar to unfamiliar contexts and to go from closed questions to more open-ended questions.

The most interesting aspect of the problem was that the results that emerged supported my hypothesis, agreeing with the idea that familiar contexts allow learners to access the necessary mathematics needed to reason and reflect on a problem. The findings brought out the aspect of open/closed problems that added a new dimension to the study, and it has made me analyse my learners' results and responses from a different perspective.

The most challenging aspect of the problem came from the grounded analysis, and the creation of categories and levels that would include all the learners' responses. Through analysing every piece of work, I created the categories and levels by making comparisons with all the responses. The work took time and required concentration and countless tweaking of categories.

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SUMMARY TABLE

EXAMINER'S COMMENTS THAT HAVE BEEN ADDRESSED	CHANGES MADE
Track changes	All track changes have been removed
Typographical and grammatical errors	According to the recommendations made by the examiner, the typographical and grammatical errors have been edited.
Notion of distance and familiarity – comment made by examiner 2	On page 29, written in italics, this issue has been addressed.
References	References have been placed in alphabetical order.
Adding LPG and CAPS to the references	The references for LPG and CAPS have been included
Distinguishing between Mathematical Literacy (ML) and mathematical literacy	This was checked in light of the comments made by examiner 2